ISSUES WITH PHASE SPACE CHARACTERIZATION OF LASER-PLASMA GENERATED ELECTRON BEAMS

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A brief tale
Diagnostics

- Point of view: electron beam diagnostics.
- We are looking not for a proof of principle experiment but for standard diagnostics:
  - Self consistent
  - Simple (Grandmother proof)
  - Reliable and hopefully simple to implement

- Old sentence: an accelerator is just as good as its diagnostics
Plasma acceleration

- We’ll talk mainly about LWFA because there is much more work already done
- Some concepts we can extend also to PWFA

- The main problems in using conventional diagnostics
  - Energy spread
  - Angular spread
Importance of RMS emittance

Even when the phase-space area is zero, if the distribution lies on a curved line its rms emittance is not zero.
RMS emittance is not an invariant for Hamiltonian with non-linear terms.
Geometrical vs Normalized

\[ \varepsilon_n^2 = \langle x^2 \rangle \langle \beta^2 \gamma^2 x'^2 \rangle - \langle x \beta \gamma x' \rangle \]

\[ \sigma_E^2 = \frac{\langle \beta^2 \gamma^2 \rangle - \langle \beta \gamma \rangle^2}{\langle \gamma \rangle^2} \]

\[ \varepsilon_n^2 = \langle \gamma \rangle^2 \sigma_{\varepsilon}^2 \langle x^2 \rangle \langle x'^2 \rangle + \]

\[ + \langle \beta \gamma \rangle^2 \left( \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2 \right) \]

For the accelerator community the normalized emittance is one of the main parameter because it is constant.

For such a beam, due to the large energy spread and huge angular divergence, it is not true anymore.

\[ \varepsilon_n^2 = \left\langle \gamma \right\rangle^2 \left( \sigma_s^2 \sigma_x^2 \sigma_x' + \varepsilon^2 \right) \]

\[ \sigma_x(s) \approx \sigma_x s \]

To measure the emittance for a space charge dominated beam, the used technique is the well-known 1-D pepper-pot

The emittance can be reconstructed from the second momentum of the distribution

\[ \varepsilon = \sqrt{\langle x'^2 \rangle - \langle xx' \rangle^2} \]

Design issues

- The contribution of the slit width to the size of the beamlet profile should be negligible.
- The material thickness (usually tungsten) must be long enough to stop or heavily scatter beam at large angle (critical issue at high energy).
- The angular acceptance of the slit cannot be smaller of the expected angular divergence of the beam.

\[
\sigma = \sqrt{(L \cdot \sigma')^2 + \left(\frac{d^2}{12}\right)}
\]

\[
L \gg \frac{d}{\sigma' \cdot \sqrt{12}}
\]

\[
l < \frac{d}{2\sigma'}
\]
Holes machining

- Holes array have been successfully produced.
- The thickness of the material can be as large as 100 times the hole diameter.

High energy pepper pot

- In principle can operate also at moderate to high energy (500 MeV-1 Gev)
- Length 50 mm, slit 500 μm, spaced 2 mm

N. Delerue and al. “TRANSVERSE EMITTANCE MEASUREMENT AT HIGH ENERGY USING LONG PEPPER-POT”, Proceedings of IPAC’10, Kyoto, Japan MOPE078
Looking for intrinsic limit of this technique for LWFA beams

- No considerations about
  - S/N ratio
  - Detector
  - Multiple scattering
  - Background
- Mask thickness neglected
Trace spaces

- All beams have $\varepsilon_n = 1$ mm-mrad
- $z=0.6$ m
- $\beta=0.1$ m means 10 $\mu$m on the source
- $\beta=0.001$ m means 1 $\mu$m on the source
No problems

\[ D_2 = 0.5 \text{m} \]
\[ \% \text{error} \sim 1\% \]
5 slits
50 \(\mu\text{m}\) size
500 \(\mu\text{m}\) distance
\[ D_1 = 0.6 \text{m} \]

\[ D_2 = 2 \text{m} \]
\[ \% \text{error} = 37\% \]
11 slits
25 \(\mu\text{m}\) size
50 \(\mu\text{m}\) distance
\[ D_1 = 0.6 \text{m} \]

\[ 5 \text{ MeV} \quad \beta = 1 \text{ m} \]
\[ 500 \text{ MeV} \quad \beta = 0.1 \text{ m} \]

Everything roughly optimized in to minimize the error and to use all the particles
No chances for $\beta=0.001\,\text{m}$

- D2=2m
- %error>1000%
- 31 slits
- 50 $\mu$m size
- 100 $\mu$m distance
- D1 0.6 m

- The phase space is so thin that the sampling is very inefficient especially in angle
This is a very good paper, well documented, a lot of details, except for the definition of the normalized emittance.

- Energy 125 MeV, energy spread 1%
- 125 μm mask thick
- Charge in the order of few pC
- Normalized emittance in the order of mm-mrad
Experimental setup

- 25 μm diameter
- 150 μm spaced
- Assuming $\beta=0.025$ and neglecting any other source of noise the error coming from undersampling is about 47% in my calculation
- Just increasing the drift up to 2 meter would reduce it to 27%
Multiple screens

\[ \sigma_{i,11} = C_i^2 \sigma_{11} + 2S_i C_i \sigma_{12} + S_i^2 \sigma_{22} \]

- There are 3 unknown quantities
- \( \sigma_{i,11} \) is the rms beam size squared
- \( C_i \) and \( S_i \) are the element of the transport matrix
- We need 3 measurements in 3 different positions to evaluate the emittance
Multiple OTR monitor?

- C. Thomas, N. Delerue and R. Bartolini “Single shot transverse emittance measurement from OTR screens in a drift transport section”, 2011 JINST 6 P07004

- In their case (3GeV) the multiple scattering is not a factor for thin (5 μm) screens
- It is possible to produce even 1 μm aluminum screen
- A waist in the drift region is a must!
- This system seems not feasible for beams with energy in the range of hundreds of MeV
Betatron radiation


\[ \lambda_b = \lambda_p \sqrt{2\gamma} \propto \sqrt{1/n_e} \]
Betatron spectroscopy


- 400 MeV energy with a rms energy spread of less than 5% and 1 mrad divergence from a plasma density of $5 \times 10^{18} \text{cm}^3$
\( \sigma \sigma' \gamma \Delta \gamma \) at the same time

- S. Kneip and al., PRST-AB 15, 021302 (2012)

Source size by Fresnel diffraction

Energy, energy spread and divergence behind the dipole
Deflecting cavity

\[ \Delta x'(z) = \frac{eV_0}{pc} \sin (kz + \varphi) \approx \frac{eV_0}{p_z c} \left[ \frac{2\pi}{\lambda} z \cos \varphi + \sin \varphi \right] \]

\[ \Delta x'_{\text{RFD}} \gg \Delta x'_{\text{beam}} \]

- In a S band deflector with \( V_0 = 2 \text{ MV} \) and bunch length \( \sim 100 \text{ fs} \) \( \Delta x' \sim 37 \text{ urad} \)
- C-band can have \( V_0 = 10 \text{ MV} \) with shorter wavelength resulting in \( \Delta x' \sim 370 \text{ urad} \)!
- The RFD can be used with a quadrupole to focus at least in the vertical plane -> limit to the energy spread.
Quadrupole scan

- Changing the strength of a magnetic lens is possible to measure the beam size.
- With at least 3 different measurements it is possible to retrieve the elements of the sigma matrix that are related with the emittance.
- Multi shot measurement.

\[ \sigma_{11} = C^2(k)\sigma_{11} + 2C(k)S(k)\sigma_{12} + S^2(k)\sigma_{22} \]
Chromatic effects

Assuming the particle energy uncorrelated from its transverse position/divergence:

$$\varepsilon_1^2 = \langle x_1'^2 \rangle \langle x_1'^2 \rangle - \langle x_1 x'_1 \rangle^2 = \varepsilon_0^2 + (kl)^2 \sigma_x^4 \sigma_y^2 = f(\varepsilon_0, \sigma_y, \sigma_x)$$

CASE 1: Moderate spot size ≈ 0.3 mm
CASE 2: Large spot size ≈ 1.7 mm

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<td>$\varepsilon_{nx}$ (mm-mrad)</td>
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<td>$\varepsilon_{ny}$ (mm-mrad)</td>
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From the experience at SPARC, we learnt that a 1.7 mm spot size at the quadrupole, with 1% energy spread, produces an error of 50% on the emittance.

A new kind of Quadscan

- R. Weingartner and al., PRST-AB 15, 111302 (2012),
Conclusions?

- Conventional diagnostic are sometimes not adequate, mainly due to the energy spread and the large angular divergence.
- The same meaning of normalized emittance must be revised.
- Pepper pot is not adequate for strongly correlated beams.
- Interesting techniques has been tested to measure the beam emittance and the longitudinal properties.
- Anyway large energy spread (>few%) seems to be an ‘hic sunt leones’ for reliable beam measurements.
Many thanks to...