
Petr Anisimov, Bruce Carlsten, Kip Bishofberger, Leanne Duffy, John Lewellen, Quinn Marksteiner, Steve Russell, Nikolai Yampolsky

Los Alamos National Laboratory

Robert Ryne

Lawrence Berkeley National Laboratory
Outline

• Review of emittance partitioning

• Example of emittance partitioning as an alternative to the laser heater in the LCLS XFEL (heat the beam with a ~ 20 keV energy spread)

• Longitudinal brightness and MaRIE requirements

• Alternative MaRIE concept – laser micro-bunching

• CSR effects
Eigen-emittance concept can be used to control phase space partitioning

• Let $\sigma$ denote the beam second moment matrix

\[
\sigma = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0 \\
\end{pmatrix}
\]

• The eigenvalues of $J\sigma$ are called eigen-emittances

• Eigen-emittances are invariant under all linear symplectic transformations, which include all ensemble electron beam evolution in an accelerator

  however, the eigen-emittances can be exchanged among the $x$-$p_x$, $y$-$p_y$, $z$-$p_z$ phase planes

• We can control the formation of the eigen-emittances by controlling correlations when the beam is generated (demonstrated in Flat-Beam Transforms (FBTs))

• We recover the eigen-emittances as the beam rms emittances when all correlations are removed
Easy to write 4-D eigen-emittances

Can find the eigen-emittances using the conservation of the 4-D determinant and of the "Raj" trace:

\[ -\frac{1}{2} Tr(J\sigma J\sigma) \]

We can always make beam waists, eigen-emittances are then:

(6D solution doesn’t have simple form)

where:

\[ \varepsilon_{eig,\pm}^2 = U \pm V \]

\[ U = \frac{1}{2} \left( \bar{\sigma}_1^2 \bar{\sigma}_2^2 + \bar{\sigma}_3^2 \bar{\sigma}_4^2 - 2BE + 2FD \right) \]

\[ V^2 = \frac{1}{4} \left( \bar{\sigma}_1^2 \bar{\sigma}_2^2 + \bar{\sigma}_3^2 \bar{\sigma}_4^2 - 2BE + 2FD \right)^2 - \left( \bar{\sigma}_1^2 \bar{\sigma}_2 \bar{\sigma}_3 \bar{\sigma}_4 - F^2 \bar{\sigma}_1 \bar{\sigma}_3 \bar{\sigma}_4 - E \bar{\sigma}_1 \bar{\sigma}_3 \bar{\sigma}_2 \right) \]
The flat beam transform (FBT) is an example

![FBT Diagram]

Observed emittances:

$$\varepsilon_{beam} = \sqrt{\varepsilon_0^2 + L^2}$$

$$L = \frac{e|B_{cath}|R_{cath}^2}{8\gamma\beta_{cm}} = |\alpha|\sigma_x^2 = \frac{1}{2}\langle xy' - yx' \rangle$$

FBT is protected from nonlinearities by symmetry and conservation of canonical angular momentum

You can always define intrinsic emittances so the FBT equations hold:

$$\varepsilon_0 = \sqrt{\varepsilon_{beam}^2 - L^2}$$

These are always zero

$$\sigma_{XY} = \begin{pmatrix} \langle xy \rangle & \langle xy' \rangle \\ \langle x'y \rangle & \langle x'y' \rangle \end{pmatrix}$$

Eigen-emittances:

$$\varepsilon_{eig,-} = \frac{\varepsilon_0^2}{2L}$$

$$L^2 \gg \varepsilon_0^2$$

$$\varepsilon_{eig,+} = 2L$$

Nothing special about x-y correlation

Can do x-z FBTs with either RF deflecting cavities or doglegs if you can impose the x-z correlation
What does symplectic mean in an rms sense?

- Lorentz force law follows from a Hamiltonian:

\[ H = c \sqrt{\left( \widetilde{p}_c - q\widetilde{A}(\mathbf{r}, t) \right)^2 + m^2 c^2 + q\phi(\mathbf{r}, t)} \]

- All electrodynamic motion satisfies Liouville’s theorem and in general leads to a nonlinear symplectic map \( \mathcal{M}(\mathbf{x}_f) = \mathcal{M}(\mathbf{x}_i) \)

- The linear part of the transformation (from quadratic part of the Hamiltonian obtained by expanding around a trajectory), satisfies

\[ J_6 = R^T J_6 R \]

- If the Hamiltonian is higher order in phase space variables, rms eigen-emittances are no longer necessarily conserved because

\[ \sigma_f \neq R\sigma_i R^T \]
Magnetized photoinjector and nonsymplectic element (from a linear perspective)

1. Start with round beam at cathode (0.7/0.7/5 μm for 500 pC)
2. FBT in the usual way gives 1.6/0.3/5 μm
3. Can use ISR from an undulator or wedge-shaped foil to generate correlation between x and energy
4. Use a dogleg to recover the eigen-emittances 0.3/0.3/25 μm for 500 pC
5. It doesn’t quite work this way because of the RF curvature in the longitudinal phase space, but we can get something like 0.2/0.2/15 μm for 250 pC

We’re going to do a relevant LCLS example, where we use emittance partitioning as an alternative to laser heating (used to suppress the microbunch instability), to increase the slice energy spread from 2 keV to 20 keV.
Eigen-emittances if there is an x-energy correlation

We need some formulas, which we can derive from the earlier material.

If there is an initial rms energy spread of \( \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{initial}} \) and longitudinal unnormalized emittance of \( \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{initial}} \sigma_z \) an induced energy slew of \( \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{slew}} \) leads to these new eigen-emittances:

\[
\varepsilon_{n+} = \gamma \beta \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{slew}} \sigma_z
\]

\[
\varepsilon_{n-} = \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{initial}} \varepsilon_{n,x}
\]

If making the slew also induces an emittance growth and an additional energy spread, the lower eigen-emittance changes to:

\[
\varepsilon_{n-} = \gamma \beta \left( \left( \frac{\Delta \gamma}{\gamma} \right)^2_{\text{ind}} + \left( \frac{\Delta \gamma}{\gamma} \right)^2_{\text{int}} \right)^{1/2}
\]

\[
\varepsilon_{n-} = \gamma \beta \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{slew}} \left( \varepsilon^2_{x,\text{ind}} + \varepsilon^2_{x,\text{int}} \right)^{1/2}
\]
We can use a dogleg to recover the eigen-emittances

\[
M_{\text{dog}} = \begin{pmatrix}
1 & L & 0 & \eta \\
0 & 1 & 0 & 0 \\
0 & \eta & 1 & \xi \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
L = S_1 \frac{1}{\cos^3 \theta_0} + 2 \frac{D}{\cos \theta_0} + S_2
\]

\[
\eta = S_1 \frac{\sin \theta_0}{\cos^2 \theta_0} + 2 \frac{D}{\sin \theta_0} \left( \frac{1}{\cos \theta_0} - 1 \right)
\]

\[
\xi = S_1 \frac{\sin^2 \theta_0}{\cos^3 \theta_0} + 2 \frac{D}{\sin \theta_0} \left( \frac{\sin \theta_0}{\cos \theta_0} - \theta_0 \right)
\]

If we pick the right dispersion \( \eta = -1/(\Delta \gamma / \gamma)_{\text{slew}} \) we just about recover the eigen-emittances

\[
\epsilon_x^2 = \chi^2 \epsilon_{x0}^2 \\
\epsilon_z^2 = \epsilon_{x0}^2 + \epsilon_{z0}^2 + \eta^2 \langle x_0^2 \rangle \langle z_0^2 \rangle + \frac{1}{\eta^2} \langle x_0^2 \rangle \langle z_0^2 \rangle
\]

\[
\chi = \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{initial}} \left( \frac{\Delta \gamma}{\gamma} \right)_{\text{slew}}
\]

which is okay (the product of the emittances has only grown a slight bit):

\[
\epsilon_x^2 \epsilon_z^2 = \epsilon_{x0}^2 \epsilon_{z0}^2 + \chi^2 (\epsilon_{x0}^2 + \epsilon_{z0}^2) \epsilon_{x0}^2 + \chi^4 \epsilon_{x0}^2
\]
We can get the x-energy correlation with a canted undulator

Energy loss in an undulator:
\[ \Delta E[\text{MeV}] = 6.3310^{-4} E^2[\text{GeV}] B^2[T] L[\text{m}] \]
or
\[ \frac{(\Delta \gamma)_{ISR}}{\gamma} = 6.4310^{-10} \gamma B_{\text{rms}}^2[T] L[\text{m}] \]

Energy diffusion in an undulator:
\[ \frac{(\Delta \gamma)_{\text{diff}}}{\gamma} = \sqrt{\frac{55}{48\sqrt{3}}} \sqrt{\frac{\hbar e^5}{4\pi\varepsilon_0 m^5 c^6}} B^{3/2} \gamma L^{1/2} = 3.8010^{-10} \gamma B^{3/2}[T] L^{1/2}[\text{m}] \]

K = 0.934 \lambda_u[\text{cm}] B[T] < 1
\[ = 4.1610^{-10} \gamma B_{\text{rms}}^{3/2}[T] L^{1/2}[\text{m}] \]

Ratio of energy slew to energy diffusion:
\[ \frac{(\Delta \gamma)_{\text{diff}}}{(\Delta \gamma)_{ISR}} = \frac{0.65}{B_{\text{rms}}^{1/2}[T] L^{1/2}[\text{m}]} \]

We'll use a 15 T field over 15 m in the following example (we're at 135 MeV), leads to a ratio of about 1:25

where we need this peak magnetic field:
\[ B_{\text{rms, max}}[T] = \sqrt{\frac{(\Delta \gamma)_{\text{ISR}}}{\gamma}}_{\text{max}} = 6.4310^{-10} L[\text{m}] \gamma \]
Baseline LCLS 250 pC case has emittances $\sim 0.53 \, \mu m$

This baseline case provides an emittance for comparison

**Thermal emittance is $\sim 0.3 \, \mu m$, slice emittance $\sim 0.47 \, \mu m$ – very good design**
First stage is a flat-beam transform

We move the focus solenoid 8.55 cm towards the cathode, which generates about 140 G at the cathode.

Thermal emittance is about 0.42 μm

We can estimate the intrinsic emittances from $\varepsilon_{beam}^2 = \varepsilon_0^2 + (\gamma \beta L)^2$

where $\gamma \beta L = \frac{eB_{cath}}{2cm} x_{rms}^2 = 0.62 \mu m$

which leads to intrinsic emittances of 0.72 μm (0.65 μm from the product of the eigen-emittances)

We’ve generally seen that the intrinsic emittances decrease in an FBT
We linearize the longitudinal phase space at 135 MeV before we do the FBT diagonalization.

LCLS configuration except for moving the solenoid forward

Longitudinal and transverse phase spaces before linearization (rms x/y-emittance is 1.64/0.26 μm for 100% of beam, 1.61/0.20 μm for the central 51% of beam)

Linearized longitudinal phase space after linearization – central 51% of beam has about a 2.1 keV rms energy spread (+/- 0.71 mm from center)
The energy slew allows us to compress the horizontal phase space in the dogleg (idealized case)

After the canted undulator – an x-energy correlation has been generated (initial x-emittance is 1.61 μm for 50% of beam, rms energy spread is 19.5 keV, slice energy spread is 2.1 keV)

The dogleg rotates the x-distribution onto itself (final x-emittance is 0.18 μm, slice and rms energy spread are 19.5 keV)

The final y-emittance is also 0.18 μm, from the loss of the additional 1% of the electrons
We still get essentially the same compression and emittance partitioning with realistic simulations

Space-charge effects at 135 MeV grow the horizontal emittance to 0.21 μm (from 0.18 μm)

Energy diffusion from an 15.2 T/15 m magnet grows this to 0.25 μm, essentially no growth for a 5 T/100 m racetrack magnet configuration.
Transversely varying inverse Compton scattering (ICS) is like a canted undulator, but it’s in a non-undulator regime for these parameters and doesn’t work.

This kind of scheme smears out the difference between slice and projected transverse emittances.

The transverse emittance in the bunch head/tail (the stuff we filtered out in the plots) has grown significantly.

Future XFELs (for us MaRIE in particular) will have limited energy-spread headroom. In that case, a reversible laser heater would be required instead.

There is some room to increase the energy spread headroom. The potential depression across the beam \( \Delta \phi = \frac{I}{4 \pi \varepsilon_0 \beta c} \) for this case is about 1.6 keV (rms is less).
Wedge-shaped foil should also allow us to achieve $\sim 0.2 \mu m$

Simulations indicate that we need to lose at least 75% of particles to maintain the partitioned emittance:

$$N < \sqrt{\frac{2}{\text{slew}} + \frac{2}{\text{ind}}} \sqrt{\frac{2}{x,\text{ind}} + \frac{2}{x,\text{int}}}$$

$\varepsilon_x = 10 \mu m$

$1 \text{ nC} \rightarrow 250 \text{ pC}$

$\varepsilon_x = 0.23 \mu m$

$\varepsilon_y = 0.12 \mu m$

“slice” $\varepsilon_z \sim 6 \text{ mm}$ by $12.5 \text{ keV} = 150 \mu m$
Simulation results for pushing 800-MeV protons through a foil have been studied. This would be the first demonstration of an XZ-FBT, experiment led by Kip Bishofberger.
Specific MaRIE topics

- For MaRIE, the electrons’ energy spread is critical ($\Delta \gamma \sim 1$); we are concerned about both the microbunch instability and undulator resistive wall wake.
- For suppressing the microbunch instability in our baseline design concept, SLAC is helping us with a reversible laser heater design.
- Alternatively, we are considering laser micro-bunching the beam instead of using a BC2.
- BC1, BC2 placement driven by IBS.
- We are concerned with CSR noise and microbunch enhancement.
Energy spread considerations

- Nominal 2 keV slice energy spread from photoinjector $\times 250$ (factor of compression BC1 times BC2) = 500 keV ($\Delta \gamma \sim 1$)

- We care about $\sim 1$ keV at 250 MeV ($4 \times 10^{-6}$ level), makes us re-evaluate wakes, space harmonics (space-charge is okay)

- 20 keV (laser heater) $\times 250$ (compression) = 5 MeV ($\Delta \gamma \sim 10$ – bad)

- IBS scales as $\delta \gamma \sim \frac{\Delta \gamma I}{\gamma' x_{rms} \varepsilon_{norm}}$ can be a few keV at 250 MeV!

- IBS looks like an important driver for BC1, BC2 placement, this effect will drive them to as low energy as possible
A longer bunch helps mitigate the undulator resistive wall wake

Resistive wake of 30 fs, 3 kA current dist.

Wake of 150 fs, 600 A current dist.

Wake of Laser bunched beam with 1 um, 600 A DC, 3.5 kA peak current dist.
Walk through the laser micro-bunching idea (1/4)

- Initial look shows promise
- Only makes sense for MaRIE parameters
- Not S2E model yet, but starting with Russell’s OPAL photoinjector simulations (about a keV slice energy spread)
- Unoptimized simulations indicate lower slice energy spread (maybe), reduced risk of microbunch instability/undulator resistive wall wake
Walk through the laser micro-bunching idea (2/4)

- Laser-chicane compressor at 1 GeV as nominal compromise for IBS and emittance growth

Effect of IBS at 1 GeV

~ 5 MeV slew before BC1
- need to operate only 10° off crest for rest of accelerator
Walk through the laser micro-bunching idea (3/4)

- Laser modulator followed by very small chicane for compression of micro-bunches all at 1 GeV
Walk through the laser micro-bunching idea (4/4)

- $\Delta \gamma = 1$ for laser-bunched simulations at right, $\Delta \gamma = 0.3$ below.

We believe we can regain a factor of a few more photons with higher harmonic compression and multi-pass lasing.
CSR noise is enhanced at higher energies

- CSR3D w/ multi-level parallelism to perform large # of simulations to study statistical properties of shot noise fluctuations.
- rms width appears to grow as $\gamma^2$
- Correlation length scales as $\gamma^{-3}$
- Induced energy spread looks ok so far ($\sim$ keV at BC2 compared to $\sim$ 10 keV from quantum diffusion)

<table>
<thead>
<tr>
<th>E (MeV)</th>
<th>Mean (MV/m)</th>
<th>$\sigma_{E_z,rad}$ (MV/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.155</td>
<td>0.00033</td>
</tr>
<tr>
<td>100</td>
<td>-2.32</td>
<td>0.011</td>
</tr>
<tr>
<td>1000</td>
<td>-2.30</td>
<td>1.105</td>
</tr>
</tbody>
</table>

Histograms of $E_z,rad$ at bunch centroid (at fixed $\rho=1$m), deposited in 64 bins, obtained from 12000 simulations, each w/ 624M particles
Squeezing the bunch in the y-direction (i.e. perpendicular to the bend plane) enhances the CSR field in a microbunched beam.

Longitudinal CSR field for a 100 MeV microbunched beam for 4 cases: ball, ball squeezed in x, ball squeezed in y, ball squeezed in x & y.
2D CSR model needs to capture wavelength, energy enhancements due to microbunching

- A bunch with a microstructure can have its fields enhanced by ~ 10x-100x depending on the energy, the microstructure, and the wavelength
- No adequate theory to predict the wavelength at which CSR has maximum enhancement, except in some regimes