1. INTRODUCTION

The generation of coherent intense soft-x-ray radiation would allow one to perform new and unique experiments in such fields as biology, chemistry, and physics. The possibilities offered for soft-x-ray microscopy and holography would be particularly interesting.

In this paper, we describe a system capable of providing coherent radiation with peak and average powers of the order of hundreds of megawatts and hundreds of milliwatts, respectively. Larger peak powers, of the order of a gigawatt, can be expected for UV radiation with λ in the range of 500-2000 Å. We discuss the physical principles of these systems and give examples of how they might be built.

2. FREE-ELECTRON LASER EQUATIONS

Following the work of other authors, we write the free-electron laser (FEL) equations using the phase and the energy as electron variables and use the slowly varying amplitude and phase approximation for the radiation field. These equations can be written in a general form including the effects of space-charge fields and the higher harmonics of the radiation field. To simplify our discussion, we neglect these terms and use the results of Ref. 4 to evaluate their effects. Our notation is the following:  

- z is the electron-beam and the electromagnetic-wave direction of propagation; x and y are the transverse coordinates; B_0 is the undulator magnetic field (we use a helical undulator for simplicity), and λ_0 and N_w are its period and length in number of periods, respectively; λ is the wavelength of the radiation field; γ is the electron energy in units of m_e c^2; β_z ≈ 1 is the longitudinal electron velocity and β_{⊥} = K/γ is the amplitude of the transverse velocity; the electron phase relative to the electromagnetic wave φ is related to z and t by φ = 2πz/λ_0 + 2π(z - ct)/λ; the resonant energy γ_R is related to λ_0, λ, and K by γ_R^2 = λ_0(1 + K^2)/2λ; the undulator frequency ω_0 is ω_0 = 2πcβ_z/λ_0; the amplitude E_0 and phase θ_0 of the radiation field are combined to yield a complex amplitude α = iE_0e^{iθ_0}.

To write the FEL equations, it is convenient to use a set of normalized variables and to introduce some quantities to characterize the beam properties. We will use the relativistic beam-plasma frequency

\[ \Omega_p = \left( \frac{4πε_0 c^2}{\langle γ_0 \rangle^3} \right)^{1/2}, \]  

where \langle γ_0 \rangle is the average value of the initial electron energy and r_e the electron classical radius; we introduce also the quantities

\[ \rho = \frac{K}{4} \frac{\langle γ_0 \rangle^2 \Omega_p^{2/3}}{ω_0}, \]  

and

\[ φ_0 = ω_0 \left( 1 - \frac{γ_R^2}{⟨ γ_0 ⟩^2} \right), \]  

and a normalized time

\[ τ = 2ω_0ρ \frac{γ_R}{⟨ γ_0 ⟩^2} t. \]  

Using these definitions, we can construct a set of dimensionless variables

\[ ψ = φ - φ_0 t, \]  

Received July 27, 1984; accepted September 13, 1984

An electron beam can be made to interact with an undulator magnet so that a collective unstable mode is excited. In this mode, the beam generates coherent radiation whose wavelength is determined by the undulator period and the electron energy. By proper choice of the electron-beam energy, energy dispersion, and density, one can obtain coherent radiation in the soft-x-ray region with peak and average power of the order of hundreds of megawatts and hundreds of milliwatts, respectively. Larger peak powers, of the order of a gigawatt, can be expected for UV radiation with λ in the range of 500-2000 Å. We discuss the physical principles of these systems and give examples of how they might be built.
and write the FEL equations as

\[
\begin{align*}
\psi_i &= \frac{1}{2\rho} \left( 1 - \frac{1}{\rho^2} \right) i = 1, 2, \ldots, N, \\
\Gamma_i &= \frac{1}{\rho} \left( \frac{\Delta e^{i\psi_i}}{\Gamma_i} + \text{c.c.} \right), \\
\dot{A} &= i\delta A + \frac{1}{\rho} \left( e^{-i\psi_i} \right),
\end{align*}
\]

with

\[
\delta = \frac{\Delta}{\rho},
\]

where \(\Delta\) is the detuning parameter

\[
\Delta = \frac{\gamma_0^2 - \gamma_R^2}{2\gamma_R^2}.
\]

The dot indicates differentiation with respect to \(\tau\). The angle brackets indicate an average over the particle initial phases, i.e., \(\langle \cdot \rangle = (1/N)\sum_i\), where \(N\) is the number of particles.

From these equations, we can show that the quantity

\[
H = \langle \Gamma \rangle + |A|^2
\]

is an invariant. In terms of laboratory variables, this can be written as

\[
H = mc^2 n_0(\gamma) + \frac{E_0}{4\pi} = \text{constant},
\]

which is seen to be the conservation of energy relation for the electron-beam–radiation-field system. It is also convenient, using Eq. (6), to rewrite Eq. (9) as

\[
\gamma - \langle \gamma_0 \rangle = \rho|A|^2 - |A_0|^2,
\]

which relates the change in the field amplitude \(A\) directly to the average change in electron energy. One can see from Eq. (11) that, assuming that \(|A_0| < |A|\), the quantity \(\rho|A|^2\) measures the efficiency of energy transfer from the electron beam to the radiation field.

In integrating the FEL equations, the maximum time is defined by the undulator length \(t_{\text{max}} = N_w \lambda_0/c\). In terms of the scaled time \(\tau\) this becomes

\[
\tau_{\text{MAX}} = 4\pi \rho \left( \frac{\gamma_R}{\langle \gamma_0 \rangle} \right)^2 N_w.
\]

3. THE FREE-ELECTRON-LASER COLLECTIVE INSTABILITY AND COHERENT-RADIATION GENERATION

The system of Eqs. (8a)–(8c) has been discussed in Refs. 2 and 3, where it has been shown that for \(\delta < \delta_{bh} \approx 1.9\) the system is unstable and the field amplitude \(A\) grows exponentially. Both the radiation field and the beam bunching grow exponentially. We can characterize the bunching by the parameter \(b = |\langle e^{-i\psi} \rangle|\). The nonlinear regime and the saturation that follow the initial exponential growth have also been studied in these papers.

In this paper, we discuss a collectively unstable system using the parameters that apply to an electron beam obtained from a storage ring.

We assume that the initial field amplitude is zero, and we introduce an initial noise in the electron phase distribution so that the initial value of \(b = |\langle e^{i\phi} \rangle|\) is \(1/\sqrt{N}\), \(N\) being the number of electrons in one radiation wavelength. In Fig. 1, we show the evolution of the field amplitude \(|A|\) versus \(\tau\) for different values of the initial electron-beam rms energy spread \(\sigma_{\text{e0}}\) and for \(\delta = 0\). One can see that, for \(\sigma_{\text{e0}} < \rho\), the field amplitude \(|A_0|\) reaches a value of the order of unity, so that, from Eq. (11), we have an energy-transfer efficiency of the order of \(\rho\), i.e., at the peak of \(|A_0|\) we have transferred a fraction \(\rho\) of the beam energy to the radiation field.

Figure 2 shows the evolution of the rms electron-beam energy spread for the same values of \(\sigma_{\text{e0}}\) as in Fig. 1. One can see that, when the field peaks, the energy spread becomes of the order of \(\rho\). We assume that the initial field amplitude is zero, and we introduce an initial noise in the electron phase distribution so that the initial value of \(\sigma_{\text{e0}} = 0.1\rho\), \(\sigma_{\text{e0}} = 0.75\rho\), \(\sigma_{\text{e0}} = \rho\).

The time needed to reach the peak can be seen from Figs. 1 and 2 to be \(\tau \approx 10\). Assuming that \(\gamma_0 = \gamma_R\), we can see from Eq. (12) that to reach the peak we need an undulator with a number of periods \(N_w \approx 1/\rho\).
Let us summarize the results of this section:

1. The electron-beam undulator-magnet radiation-field system is unstable if \( \beta < \beta_{th} \), and both the field amplitude \( |A| \) and the beam bunching \( b \) grow exponentially up to a saturation level, where \( |A| \sim 1 \) and \( b \sim 1 \).

2. If the system's initial conditions are \( A_0 = 0, b_0 \), determined by noise, \( \sigma_0 < \rho \), then the electron beam will transfer a fraction \( \rho \) of its energy in a number of undulator periods of the order of \( 1/\rho \).

3. After traversing the undulator, we have \( |A| \approx 1, b \approx 1 \) and \( \sigma \sim \rho \).

### 4. ELECTRON-BEAM-UNDULATOR SYSTEM

As we wish to discuss the operation of a FEL over a large wavelength range (30–2000 Å), we consider operating the storage ring at energies ranging from 300 to 500 MeV. In addition, we consider three undulator designs: a 5-mm-period undulator for \( \lambda \) in the range 90–100 Å, a 1-cm-period undulator for \( \lambda \) in the range 100–250 Å, and one with \( \lambda_0 = 2.5 \) cm for \( \lambda \) in the range 500–2000 Å.

To calculate the undulator properties, we assume that the undulator is of the hybrid (permanent-magnet and iron) type and calculate the magnetic field from

\[
B_0 = 3.33 \exp\left(-5.47 \frac{g}{\lambda_0} + 1.8 \frac{g^2}{\lambda_0^2}\right) T, \tag{13}
\]

where \( \lambda_0 \) and \( g \) are the period and the gap, respectively. A complete listing of the undulator data can be found in Table 1. The output-radiation wavelengths for the three undulators are signified by \( o \)'s in Fig. 3.

The electron beam described in Table 2 can be obtained in a storage ring as we see in Section 5. However, if we tried to install the undulators described in Table 1 directly in the ring such that the electron beam would pass through the small aperture of the undulator on each revolution, it would become impossible to operate the ring. The small aperture (gap) of the undulator would lead to vanishing small-beam lifetimes resulting from collisions with undulator walls. The minimum allowable gap depends on both the ring and the undulator parameters and must be determined experimentally.

For this reason, we propose to install the undulator in a ring bypass, as shown in Fig. 4. The electron beam would normally circulate in the ring, where the effect of synchrotron-radiation damping would produce the beam properties of Table 3. About once per damping time, of the order of 50 msec for the storage ring illustrated in Table 3, the beam is taken into the bypass and focused in the undulator by a special quadrupole triplet. In going through the undulator, the electron beam produces the radiation, its energy is decreased by \( \rho E_T \), and its energy spread increases from its initial value to about \( \rho \). The beam is then taken back into the storage ring and left there for a time long enough for synchrotron-radiation damping to bring its characteristics back to their starting value. A more detailed discussion of the storage ring and the bypass system is given in Section 5.

As the electron beam circulates in the ring, it performs both vertical and horizontal oscillations (the so-called betatron oscillations). The betatron functions \( \beta_H \) and \( \beta_v \), which characterize the amplitude and the period of the betatron oscillations, are a measure of the focusing properties of the magnetic lattice. Together with the emittance \( \epsilon \), which is the area in the position angular-deviation phase space in which the beam is contained, the beta functions determine the beam size, i.e., the rms beam height \( \sigma_H = \sqrt{\epsilon \beta_H} \). The choice of the beta functions in the bypass is determined by the re-

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**Table 1. Undulator Magnets**

<table>
<thead>
<tr>
<th>Undulator Properties</th>
<th>Undulators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period, ( \lambda_0 ) (cm)</td>
<td>0.5</td>
</tr>
<tr>
<td>Gap, ( g ) (cm)</td>
<td>0.1</td>
</tr>
<tr>
<td>Pump strength, ( B_0 ) (T)</td>
<td>1.2</td>
</tr>
<tr>
<td>Undulator parameter, ( K )</td>
<td>0.56</td>
</tr>
</tbody>
</table>

**Table 2. Electron-Beam Parameters in the Bypass Section**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Electron Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, ( E ) (MeV)</td>
<td>500.00</td>
</tr>
<tr>
<td>Beta horizontal, ( \beta_x ) (m)</td>
<td>3.00</td>
</tr>
<tr>
<td>Beta vertical, ( \beta_y ) (m)</td>
<td>1.00</td>
</tr>
<tr>
<td>Coupling, ( \chi )</td>
<td>0.01</td>
</tr>
<tr>
<td>rms horizontal beam radius, ( \sigma_x ) (cm)</td>
<td>1.01E-04</td>
</tr>
<tr>
<td>rms horizontal angular spread, ( \sigma_x' ) (rad)</td>
<td>3.37E-05</td>
</tr>
<tr>
<td>rms vertical angular spread, ( \sigma_y' ) (rad)</td>
<td>5.81E-06</td>
</tr>
<tr>
<td>rms vertical beam radius, ( \sigma_y ) (m)</td>
<td>5.81E-06</td>
</tr>
</tbody>
</table>

**Fig. 3.** Plot of the output wavelength \( \lambda \) and the estimated peak-power output versus energy for (a) \( \lambda_0 = 5 \) mm, (b) \( \lambda_0 = 1 \) cm, (c) \( \lambda_0 = 2.5 \) cm. The \( \lambda \) values are signified by \( o \)'s, and the power values are given by \( O \)'s. For the power curves, a solid line (—) corresponds to \( Z = 0.1 \) Ω, a dashed line (---) to \( Z = 1 \) Ω, and a dotted–dashed line (-----) to \( Z = 10 \) Ω.
When the beam enters the bypass section, it undergoes additional focusing to increase $\rho$, as shown in Table 2.

The ring has two 10-m-long straight sections, one used for the radio-frequency system and one for the bypass switching magnets. The arcs joining the two long straight sections each have three equal periods. Each period has two dipole magnets with a focusing quadrupole between them and two quadrupole doublets on the external sides. The ring energy dispersion is controlled by the central quadrupole and is nonzero only in the dipoles and in the region between them.

The momentum compaction $\alpha = (d\varepsilon/E)/(dl/l)$ relates the change in orbit length to the relative energy deviation from the design energy $E_0$ of the ring. For a ring with this magnetic structure, the momentum compaction $\alpha$ and the horizontal emittance $\epsilon_H$ are approximately given by

$$\alpha = \frac{1}{6M} \frac{R_B}{R_{av}}$$  \hspace{1cm} (15)

$$\epsilon_H = 7.7 \times 10^{-13} \frac{\gamma^2}{M^5} \text{mrad},$$  \hspace{1cm} (16)

where $R_B$ and $R_{av}$ are the bending and the average ring radii and $M$ is the number of achromatic bends. An achromatic bend typically consists of two dipole magnets with a horizontally focusing quadrupole between them and is designed to focus all the entering electrons, regardless of energy, to the same point when they leave the bend. The vertical emittance is determined by the coupling between the horizontal and the vertical oscillation that is due to the magnet misalignment $\epsilon_V = \chi \epsilon_H$.

At zero or small current, the rms energy spread and the bunch length are determined by synchrotron radiation and are given by

$$\sigma_{\phi \theta} = 4.38 \times 10^{-7} \frac{\gamma}{\nu_s}$$ \hspace{1cm} (17)

$$\sigma_{\phi \phi} = \frac{\alpha R_{av}}{\nu_s} \sigma_{\phi \theta},$$ \hspace{1cm} (18)

where $\nu_s = \omega_s/\omega_0$ is the ring synchrotron-oscillation tune. At large currents, the microwave instability caused by the beam interaction with the broad-band high-frequency storage-ring impedance can increase the energy spread $\sigma$, and the bunch length $\sigma_p$. An increase of $\sigma_p$ reduces the value of $\rho$, while $\sigma$ increases and the condition $\sigma_p < \rho$ can be violated.

To evaluate this effect, we use the approximate condition

$$e_{p} \left| \frac{Z(n)}{n} \right| \leq 2\pi E_0 \alpha e^2$$ \hspace{1cm} for $n \geq \frac{R_{av}}{\sigma_p},$$ \hspace{1cm} (19)

where $I_p$ is the peak current related to the average bunch current $I_0$ by

5. ELECTRON STORAGE RING AND BYPASS SECTION

The storage ring that we consider is similar to those used as synchrotron-radiation sources, for instance, the National Synchrotron Light Source VUV ring. Its main characteristics are given in Table 3.

In what follows, we will make sure that the condition $\sigma_{\text{eff}} < \sigma$ is always satisfied.

In the cases that we consider in Section 5, the undulator length varies between 2 and 3 m and is determined by the condition $N_w \approx 1/\rho$. This length is also consistent with our assumption on the beta functions.

### Table 3. Storage-Ring Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Electron Beam Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, $E$ (MeV)</td>
<td>500 400 300</td>
</tr>
<tr>
<td>Gamma, $\gamma$</td>
<td>978 782 587</td>
</tr>
<tr>
<td>Bending radius, $R_B$ (m)</td>
<td>4 4 4</td>
</tr>
<tr>
<td>Average radius, $R_{av}$ (m)</td>
<td>15 15 15</td>
</tr>
<tr>
<td>Number of achromatic bends, $M$</td>
<td>6 6 6</td>
</tr>
<tr>
<td>rf voltage, $V$ (MV)</td>
<td>1 1 1</td>
</tr>
<tr>
<td>Harmonic number, $h$</td>
<td>100 100 100</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>1 1 1</td>
</tr>
<tr>
<td>Average current, $I_0$ (A)</td>
<td>0.10 0.10 0.10</td>
</tr>
<tr>
<td>Harmonic number, $h$</td>
<td>100 100 100</td>
</tr>
<tr>
<td>Average radius, $R_{av}$ (m)</td>
<td>15 15 15</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>1 1 1</td>
</tr>
<tr>
<td>Average current, $I_0$ (A)</td>
<td>0.10 0.10 0.10</td>
</tr>
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<td>100 100 100</td>
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<tr>
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<td>1 1 1</td>
</tr>
<tr>
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<td>0.10 0.10 0.10</td>
</tr>
<tr>
<td>Harmonic number, $h$</td>
<td>100 100 100</td>
</tr>
<tr>
<td>Average radius, $R_{av}$ (m)</td>
<td>15 15 15</td>
</tr>
</tbody>
</table>

### Table 4. Electron-Beam Bunch-Length and Peak-Current Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Electron Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective coupling impedance ($\Omega$)</td>
<td>0.1 1.0 10</td>
</tr>
<tr>
<td>Bunch length, $\sigma_p$ (microwave limit) (cm)</td>
<td>0.95 2.0 4.4</td>
</tr>
<tr>
<td>Peak current, $I_p$ (A)</td>
<td>397 184 86</td>
</tr>
</tbody>
</table>
\[ I_p = (2\pi)^{1/2} \frac{R_{av}}{\sigma_p} I_0 \]  
\[ |Z(n)/n| \text{ is the effective longitudinal coupling impedance of the ring.} \]

From expressions (18)-(20), the following expressions for the microwave-instability-limited bunch length and energy spread can be obtained:

\[ \sigma_p = R_{av} \left( \frac{\alpha_{fg}(\sqrt{2\pi \epsilon V_{n2}}) Z_n}{n} \right)^{1/3}, \]
\[ \sigma_i = \frac{\nu_s}{\alpha R_{av}} \sigma_p. \]

The storage-ring coupling impedance is determined by the vacuum-chamber geometry and by the bending radius in the curved section and is a quantity difficult to calculate \emph{a priori}. However, in modern storage rings, values of the order of 1 \( \Omega \) have been obtained. Since this quantity is important in determining the performance of our system, we have chosen to use in our calculations three values of \(|Z(n)/n|\), i.e., 0.1, 1, and 10 \( \Omega \). Let us notice that a 10-\( \Omega \) coupling impedance is large and is a pessimistic assumption, whereas a 1-\( \Omega \) value is realistic and has been already obtained. On the other hand, a 0.1-\( \Omega \) value would require a breakthrough in storage-ring design.

The microwave-instability-limited bunch lengths and peak currents, which depend on the value of the coupling impedance but not on the energy, are given in Table 4. The bunch lengths are typically of the order of a few centimeters, and the peak currents are in the 100-400-A range.

To test the beam for stability against transverse coherent oscillations, we have used the conditions that the coherent betatron tuning shift is smaller than the synchrotron tune; \[ \delta \nu_b = \frac{e_1}{\pi \nu E_0} Z_{T,eff} < \nu_s, \]

with the transverse coupling impedance \( Z_{T,eff} \) evaluated from the longitudinal impedance as:\[ Z_{T,eff} = 2 \left( \frac{R}{\beta} \right) \left| \frac{Z(n)}{n} \right|. \]

The ring described in Table 3 will be free from transverse instability problems provided that the effective impedance can be kept on the order of 1 \( \Omega \) or less.

As a final measure of the ring's feasibility, we compute the Touschek lifetime. The Touschek lifetime is the time in which losses resulting from Coulomb collisions between electrons in the same bunch have reduced the beam current to half of its initial value. For the range of ring parameters given in Table 3, \( \tau_t > 1 \) h.

6. RESULTS

In Fig. 5, we plot the FEL parameter \( \rho \) and the microwave-instability-limited energy spread \( \sigma_i \), versus energy for (a) \( \lambda_0 = 5 \text{ mm} \), (b) \( \lambda_0 = 1 \text{ cm} \), (c) \( \lambda_0 = 2.5 \text{ cm} \). The \( \rho \) values are signified by \( O' \)'s, and the \( \sigma_i \) values are given by \( O' \)'s. Each figure displays the \( \rho \) and \( \sigma_i \) values for three values of \( Z_{eff} = 0.1, 1.0, 10 \text{ } \Omega \). A solid line corresponds to \( Z = 0.1 \text{ } \Omega \), a dashed line to \( Z = 1 \text{ } \Omega \), and a dotted-dashed line to \( Z = 10 \text{ } \Omega \). The lines are not fitted to the points; they are drawn simply to indicate trends.

Figure 3 shows plots of the peak power versus energy for the three undulator designs. The peak power is calculated assuming that the radiation-pulse length is equal to the electron-bunch length.

7. CONCLUSIONS

Using the system described (an electron storage ring with an undulator in a special bypass section), we can obtain high-intensity coherent radiation by sending the beam through the undulator and by using the FEL collective instability to produce radiation. Compared to other systems, such as a FEL oscillator or a transverse optical klystron, this system has the advantage that it does not require mirrors to form an optical cavity or an input high-power laser to bunch the electron beam. On the other hand, by its nature, this system can produce only high-intensity short-radiation pulses with a repetition rate of the order of 10 Hz.

The storage ring needed to operate the system is characterized by a small transverse emittance. The other important ring parameter is the longitudinal coupling impedance. For a value of the order of 1 \( \Omega \), one can obtain peak powers of the order of 500 MW down to wavelengths of about 500 \( \AA \) and peak powers of 50 MW down to wavelengths of 80 \( \AA \); the power decreases sharply at lower wavelengths. If it should become possible to reduce \(|Z(n)/n|\) to 0.1 \( \Omega \), one could get peak powers of the order of 20 MW down to wavelengths of 30 \( \AA \).

One should also remember that in this paper we have concentrated our attention on the first-harmonic production only; however, from the results of Ref. 4, we know that the system will also produce higher harmonics, and this can shift down the lower limit for soft-x-ray production.

We want to emphasize that the results presented here are preliminary and that one might improve the system performance by optimizing other ring parameters, such as the momentum compaction, the radio frequency, and the radio-frequency voltage. To obtain a more complete understanding of the system, one should investigate diffraction effects on the radiation that result from the finite-beam radius and consider a three-dimensional calculation, taking into account the
electron-density variation in both the transverse and longitudinal directions.

This research was performed under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

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