PROGRESS TOWARD A SOFT X-RAY FEL *

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We review the FEL physics and obtain scaling laws for the extension of its operation to the soft X-ray region. We also discuss the properties of an electron beam needed to drive such an FEL, and the present state of the art for the beam production.

1. Introduction

The recent successes in operating FELs from the far infrared to the visible and near UV [1], together with a better understanding of the physics and technology of these systems, are leading us to explore again the possibility of using FELs to produce coherent radiation in the XUV region, at wavelength shorter than 100 nm.

In this paper we derive a set of scaling laws, which we will use to study XUV FELs, to obtain their possible performance, and, most important, to determine the characteristics of the electron beam needed to drive the FEL.

To build a XUV FEL there are two main issues that we must face: (a) to obtain mirrors with large enough reflectivity to build an optical cavity in this wavelength range; (b) to produce an electron beam with the energy, high density, and small momentum spread, required to produce large gain.

The mirror problem is complicated by the fact that the light falling on the mirror has two components. One is the light at wavelength that the FEL is amplifying; the other is the background spontaneous radiation, whose spectrum usually extends beyond this wavelength. This background radiation can damage the mirror surface and strongly reduce its reflectivity [2,3]. This is specially true for multilayered mirrors.

Although much work is being done on this problem, and it is possible that in the future we might have the required mirrors, I prefer in this paper to consider a different approach to the production of XUV radiation, which does not require optical elements. This approach uses Amplified Spontaneous Emission (ASE) to produce radiation starting from the beam noise, in a long undulator [4–6]. This system can be seen as a natural extension of synchrotron radiation production from an undulator. For a short undulator the radiation that we observe is that due to the noise in the longitudinal density distribution present in an electron beam, and has a linear dependence on the number of electrons in the beam. If the undulator is made long enough, and the beam intensity is large, this spontaneous emission starts to be amplified by the beam itself, and the output radiation will grow exponentially, until it reaches saturation. It is worthwhile to notice that this saturation level is the same as the saturation intracavity beam power in an oscillator [5].

Although the use of ASE eliminates the need to use mirrors, it requires, in order to reach saturation, a larger electron beam peak current and a longer undulator than in the case of an oscillator. Another difference is that an oscillator can be operated with a large number of electron bunches, separated by a distance equal to twice the cavity length, traversing the optical cavity, and each one amplifying the preexisting radiation, while in the ASE mode we have a single bunch traversing the undulator and interacting with its own radiation. This difference has an effect on the design of the accelerator producing the beam. For instance, if we use a linac, we have to consider the effect of the beam breakup instability in the case of many bunches, which puts a limit to the average current that one can accelerate [7]. For a high current single bunch, the main effect is due to the instantaneous wakefield, which can limit the peak current and the energy spread [7].

In what follows we will first review the basic properties of an FEL, limiting ourselves to the high gain regime, which is the one of interest in the XUV region, and in particular for the ASE. We will then derive the scaling laws for an FEL in the ASE mode of operation, and use these results to estimate the FEL performance, based on the assumption of an electron beam with given properties. In the last section we will discuss the production of the electron beam, considering two different accelerators, a storage ring and a linac.

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2. FEL physics

In this section we collect all the basic formulae describing the FEL physics, using the results and notations of refs. [4–6]. For simplicity we will use a one-dimensional model of the FEL and give conditions for it to be a good approximation of the real system.

The notations we use are the following. The light velocity is \( c \), \( r_e \) is the electron classical radius, and \( e \) is the electron charge. The electron energy, \( \gamma \), is measured in units of the electron rest energy, \( mc^2 \). The undulator period and magnetic field are \( \lambda_u \) and \( B_u \). The undulator parameter is \( K = eB_u\lambda_u/2\pi mc^2 \); \( \omega_0 = 2\pi c/\lambda_u \) is the frequency associated to the undulator period; \( \Omega_p = (4\pi e^2 n_e/\gamma)^{1/2} \) is the plasma frequency of an electron beam of particle density \( n_e \), and energy \( \gamma \). The radiation wavelength is \( \lambda \).

With these notations, and considering again for simplicity a helical undulator, we can write the FEL synchronism condition as

\[
\lambda = \left( \lambda_u/2\gamma^2 \right) (1 + K^2). \tag{1}
\]

The most important parameter describing the FEL system is the quantity [4]

\[
\rho = \left( \frac{K}{4\gamma\omega_0} \right)^{2/3}. \tag{2}
\]

This quantity gives the radiation field growth rate in the high gain limit, the energy transfer from the electron beam to the radiation, the FEL saturation length, and part of the conditions on the beam for the one-dimensional model to apply.

The growth rate for the radiation field amplitude is given by

\[
E = E_0 e^{4\pi n_b N_u \Lambda(\Lambda)}, \tag{3}
\]

where \( \Lambda \) is the solution of a cubic dispersion relation [4].

For simplicity we use an approximate form of this dispersion relation, valid for the one-dimensional model, neglecting space charge, and for \( \rho \) smaller than 0.01: \( \Lambda^3 - 1 = 0 \). Taking the root corresponding to exponential growth, we have that the exponential growth rate of the radiation field amplitude, \( E \), is given by

\[
E \approx E_0 \exp \left( 2(3)^{1/2} \pi \rho N_u \right). \tag{3'}
\]

This exponential growth rate saturates, because of electron trapping in the ponderomotive potential well [5]. The saturation is reached when the growth rate and the electron oscillation frequency in the ponderomotive potential well are about equal. In the case of ASE the undulator length needed to reach saturation is given by

\[
N_u^s \approx 1/\rho. \tag{4}
\]

The power transfer, from the electron beam to the radiation, at saturation is

\[
P_L \approx \rho P_{beam}, \tag{5}
\]

More power can be extracted tapering the undulator [8], but we will not consider this possibility in this paper.

For the one-dimensional model to apply, and the growth rate to be given by eq. (1), the electron beam driving the FEL must satisfy three conditions [5]:

(a) a limit on the beam energy spread, \( \sigma_\gamma \), to avoid a gain reduction

\[
\sigma_\gamma/\gamma < \rho, \tag{6}
\]

(b) a limit on the beam transverse emittance, \( \epsilon \), again to avoid a gain decrease

\[
\epsilon < \lambda/2\pi, \tag{7}
\]

(c) a condition for radiation diffraction effects to be negligible

\[
Z_R \approx L_G. \tag{8}
\]

The two quantities introduced in the last equation are the radiation Rayleigh range for a radius equal to the beam radius, \( a \): \( Z_R = \pi a^2/\lambda \); and the gain length, \( L_G = \lambda_u/(2(3)^{1/2}\pi \rho) \), defined as the undulator length corresponding to one e-folding length, in equation (3'). Equation (8) requires that diffraction does not take radiation out of the beam in a distance equal to the gain length: \( (\lambda/a) L_G < a \).

A more detailed description of three-dimensional effects can be obtained, following refs. [9–12], in terms of the transverse modes of the radiation produced by the electron beam. When eq. (8) is satisfied only one transverse mode, usually a Gaussian mode, is amplified, and the gain is given with good approximation by the one-dimensional theory.

All the basic physics described in these formulas, like exponential growth from noise or optical guiding, has been proved experimentally in the near or far infrared. Assuming that the same physics remain valid at shorter wavelength, we can use these formulas to design a soft X-ray FEL.

3. Scaling laws

To estimate what is the best way of designing an FEL, we want to look at the scaling laws of this system. It is convenient to rewrite \( \rho \) for given \( \lambda \) and using the beam invariants quantities, \( \epsilon_N, \epsilon_L \), transverse and longitudinal normalized rms emittances, and the longitudinal brilliance

\[
B_L = eN_c / \left( (2\pi)^{1/2} \epsilon_L \right). \tag{9}
\]

The emittances we use are defined as rms values. For the longitudinal emittance we define it as the product of the rms bunch length and the rms value of \( \gamma \).
We also assume that in the undulator \( \sigma_u \) and \( \sigma_\perp \) are related by \( \sigma_\perp = \beta_u \sigma_u \), where \( \beta_u \), the betatron oscillation wavelength, characterizes the transverse focusing in the undulator [5]. In a helical undulator the built in focusing gives \( \beta_u = (2)\gamma \lambda_u (2\pi K)^{-1} \). This focusing might not be enough to produce the desired gain, and we assume in the following that we add external quadrupoles to control \( \beta_u \) as is more convenient for us.

Using \( \epsilon_N \) and \( B_L \) and introducing the current \( I_A = ec/\gamma \), we can rewrite eq. (2) as

\[
\rho = \left[ \frac{1}{4\pi} \frac{K}{1 + K^2} \left( \frac{\sigma_u}{\gamma} \right)^{1/2} \left( \frac{2B_L}{\beta_u \epsilon_N I_A} \right)^{1/2} \right]^{3/2}. \tag{10}
\]

We can further simplify the expression for \( \rho \), and obtain a formula convenient for a first order calculation. Assuming \( \sigma_\perp/\gamma = \rho/2 \), \( \epsilon_N = \gamma \lambda/2 \pi \), \( K = 2^{1/2} \) we obtain

\[
\rho = 0.14 (B_L/I_A \beta_u)^{1/2} \gamma (\lambda)^{1/2}. \tag{11}
\]

If we can focus to \( \beta_u = 1 \) m at all energies it is important to notice the weak dependence of the FEL parameter, \( \rho \), on the radiation wavelength and, in fact, on all other parameters. The strongest dependence is on \( \gamma \), and this is the one parameter that is easy (although expensive) to increase. Since \( \rho \) decreases only weakly with wavelength, we have a good possibility of extending the FEL operation to short wavelengths: then \( \rho \) increases with \( \gamma \). The price to pay is that \( \lambda_u = \gamma^2 \lambda \) also increases with gamma, and the undulator becomes longer. Since \( N_u = 1/\rho \), \( L_u \) increases only linearly with gamma.

4. Beam production

Production of high density electron beams is of interest for FELs, for synchrotron radiation sources, and for electron–positron linear collider. One can now find many studies of this subject in the literature. More recently, it has been studied and surveyed at a workshop held in Brookhaven in 1987 [13]. This workshop looked at electron beam production using damping rings or high brightness electron guns. For FEL applications the requirements were to obtain a beam with an energy in the GeV range, a longitudinal brilliance of 200 A, and a normalized transverse rms emittance of \( 10^{-6} \) m rad, which could be used for a 1 nm FEL. In this section we will mainly discuss the results reached at this workshop.

Damping rings have problems in producing a beam with \( B_L \approx 200 \) A and \( \epsilon_N \approx 10^{-6} \) m, mainly because of the microwave instability, which becomes more important when trying to reduce the emittance, and increases the bunch length and energy spread [14].

For a typical storage ring, the way to reduce the emittance is to make the energy dispersion small, and this is achieved by using a large number of small magnets to bend the electrons. If \( N_p \) is the number of bending cells used the normalized rms transverse emittance is [14]

\[
\epsilon_N \approx \left( \frac{\gamma}{N_p} \right)^3. \tag{12}
\]

At the same time the ring becomes more and more isochronous, the revolution period is nearly the same for particles of different energies. This property is measured by the momentum compaction, \( \alpha \), which also becomes small when \( N_p \) increases [14]

\[
\alpha = 1/N_p^2. \tag{13}
\]

When the particles are more isochronous, they are more easily subject to the microwave instability, which produces an increase in the bunch length and energy spread, and hence limits the longitudinal brilliance [14]

\[
B_L \approx \alpha (\sigma_\perp/\gamma)/Z_n \tag{14}
\]

where \( Z_n \), the ring longitudinal coupling impedance, is at best on the order of one Ohm.

This relationships between \( \epsilon_N \), \( \alpha \), \( B_L \), makes it difficult to satisfy all the requirements.

As an example of low emittance storage ring, we can consider the SLAC damping ring, having an energy of 1.2 GeV, a normalized transverse emittance of \( 2 \times 10^{-5} \) m rad in both the horizontal and vertical plane, and a longitudinal brilliance of 120 A [15].

Production of small emittance, large brilliance beams from electron guns, has seen rapid progress during the past few years. Very encouraging results have been obtained using large (10 to 100 MV/m) accelerating fields near the cathode, and laser driven photocathodes for picosecond pulses, or longer pulses followed by magnetic compression to reduce space charge effects at low beam energy.

As an example we give some results obtained at Los Alamos, using a rf gun, operating at about 1.3 GHz, with an electric field on the cathode of 30 MV/m, and using a CsSb photocathode, producing a current density of about 600 A/cm². This gun has produced a beam with a normalized rms emittance of \( 5 \times 10^{-6} \) m rad, at a beam energy of about 1.1 MeV, and a longitudinal brilliance of 2000 A [16].

The limitation on the emittance and brilliance of an electron gun are due to the maximum current density obtainable from the cathode, nonlinear electromagnetic forces in the gun produced fields, and space charge forces [17]. Work is now underway in several laboratories to produce photocathodes with larger current densities. Space charge effects can be reduced either by applying a very strong electric field on the cathode surface, to accelerate the electron fast to relativistic velocities, or by producing initially a long bunch, with small longitudinal charge density, and magnetically compressing it when the beam has a large energy. The
first system can be best applied by using high rf frequency; for instance, working at 3 GHz it should be possible to push the field near to the cathode to the 100 MeV/m range. In the second case it is better to work at low frequency, or in a static field. When using low frequencies, in the few hundred MHz range, it is still possible to reduce the energy spread for given bunch length, by adding harmonic components to the RF field, to make it almost linear near the bunch phase [17]. All of this points have been discussed at the Brookhaven workshop. The conclusion reached at the workshop was that using one or the other of these techniques, it should be possible to achieve both an emittance \( \epsilon_N \approx 10^{-6} \) m, and a longitudinal brilliance \( B_L \approx 200 \) A/m. If this result is achieved it should make possible a soft X-ray FEL at around a few nanometers, in the ASE mode.

5. Examples

We will consider now an example of an FEL, in the wavelength range 100 to 2.5 nm, driven by an electron beam with a normalized transverse emittance of \( 10^{-6} \) m rad and a longitudinal brilliance of 200 A. We keep the rms bunch length and energy spread constant and equal to 1 ps and 0.1%, and change the peak current as we change the energy.

The beam energy is determined by the choice of the undulator. We consider two cases, of undulators with periods of 1 or 2 cm, and keep the undulator parameter \( K \) fixed at a value equal to 1, to maximize \( \rho \) respective to \( K \) in eq. (10). We also assume that the undulator is long enough for the FEL to reach saturation, and use eq. (4) to estimate the corresponding undulator length.

| Table 1 | Soft X-ray FEL with \( \lambda_u = 2 \) cm. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \lambda, \) nm | 100  | 50  | 10  | 5   | 2.5  |
| Energy, GeV     | 0.23 | 0.32| 0.72| 1.02| 1.44 |
| \( \rho_{0x} \times 10^3 \) | 3.2  | 2.9 | 2.2 | 1.9 | 1.7  |
| \( L_u, \) m    | 6.2  | 7.0 | 9.1 | 10.2| 11.5 |
| \( Z_R, \) m    | 0.07 | 0.10| 0.22| 0.3 | 0.44 |
| \( L_G, \) m    | 0.57 | 0.64| 0.84| 0.94| 1.05 |
| \( E_{FEL}, \) MJ | 0.26 | 0.47| 1.8 | 3.2 | 5.7  |

| Table 2 | Soft X-ray FEL with \( \lambda_u = 1 \) cm. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \lambda, \) nm | 100  | 50  | 10  | 5   | 2.5  |
| Energy, GeV     | 0.16 | 0.23| 0.51| 0.72| 1.02 |
| \( \rho_{0x} \times 10^3 \) | 2.3  | 2.0 | 1.5 | 1.4 | 1.2  |
| \( L_u, \) m    | 4.4  | 4.9 | 6.4 | 7.2 | 8.1  |
| \( Z_R, \) m    | 0.10 | 0.14| 0.31| 0.44| 0.62 |
| \( L_G, \) m    | 0.40 | 0.45| 0.59| 0.66| 0.75 |
| \( E_{FEL}, \) MJ | 0.10 | 0.16| 0.63| 1.12| 2.0  |

The results are given in tables 1 and 2. For each \( \lambda \) we give the required beam energy, the FEL parameter in units of \( 10^{-3} \), the undulator length, \( L_u \), needed to reach saturation, and the radiation pulse energy, \( E_{FEL} \), at saturation. We also give the Rayleigh range and the gain length, so that we can estimate whether diffraction effects will be important.

One can see that with the undulators considered, both systems can provide a satisfactory performance near the shortest wavelengths. At around 2.5 nm one can get, an energy per pulse of the order of a few millijoule. This number is very attractive for many applications, like for instance X-ray microscopy.

References