ELSEVIER

Nuclear Instruments and Methods in Physics Research A 445 (2000) 90–94

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

www.elsevier.nl/locate/nima

Compensation of FEL gain reduction by emittance effects in a strong focusing lattice

S. Reiche*

Deutsches Elektronen Synchrotron, Notkestraße 85, 22603 Hamburg, Germany

Abstract

As the constraint of a small transverse emittance becomes more severe, the higher the electron beam energy in an FEL. To compensate for the transverse and thus the longitudinal velocity spread, a compensation scheme has been proposed previously by Derbenev and Sessler et al., for Free Electron Lasers by introducing a correlation between the energy and the average betatron amplitude of each electron. This compensation scheme is based on a constant absolute value of the transverse velocity, a feature of the natural focusing of undulators, and does not include strong focusing of a superimposed quadrupole lattice. This paper focuses on the electron motion in a strong focusing lattice with a variation in the axial velocity. The resulting reduction of the compensation efficiency is analyzed using simulations. It is seen that the compensation scheme is not much affected if the lattice cell length is shorter than the gain length. For the results presented in this paper, the parameters of the proposed TESLA X-ray FEL have been used. © 2000 Published by Elsevier Science B.V. All rights reserved.

PACS: 41.60.Cr; 41.75.Ht

Keywords: X-Ray FEL; Emittance; Betatron oscillation; GENESIS 1.3

1. Introduction

Several Free Electron Lasers are proposed or currently under construction [1–4] to extend the wavelength range to the VUV and X-ray regime. Compared to FELs operating at a shorter wavelength, an efficient FEL performance demands an improved beam quality to keep the total length of the FEL in reasonable limits. In particular, the spread in the transverse motion degrades the synchronization of the electrons with the radiation field because the longitudinal velocity is modulated by the betatron oscillation. The FEL amplification is affected by the transverse emittance similar to a larger energy spread, resulting in an increased saturation length with reduced radiation power. This emittance effect is enlarged if the focusing strength of a quadrupole lattice is increased. The optimum focusing provides a beam size, where the enhancement of the FEL amplification by a higher electron density and the degradation by emittance effects are in balance for the FEL gain [5].

The impact of the transverse motion can be reduced if the slower longitudinal velocity for a larger betatron amplitude is compensated by a higher electron energy. This implies a correlation between the average transverse position of the electron and

* Tel.: + 49-40-8998-3448; fax: + 49-40-8998-4305.

PII: S0168-9002(00)00119-4

Table 1
Design parameters of the TESLA X-ray FEL

Electron beam	
Energy	25 GeV
Peak current	5000 A
Normalized emittance	1π mm mrad
Undulator	
Period	7 cm
Undulator parameter (rms)	3.93
Length	100 m
Quadrupole gradient	35 T/m
FEL	
Radiation wavelength	2.5 Å
FEL parameter ρ	9.4×10^{-4}
Gain length	5.9 m

its energy [6,7]. If the change in the longitudinal velocity is constant the compensation scheme will be highly efficient as it is the case for the natural focusing of the undulator with no externally applied strong focusing [8].

The natural focusing is not sufficient enough for X-ray FELs with electron beam energies above 10 GeV. A quadrupole lattice increases the electron density to shorten the gain length. Typically, the period length of the betatron oscillation and thus the modulation of the longitudinal velocity is larger than the gain length. A conditioned beam, where the correlation between energy and betatron amplitude has been applied, can only compensate the average change in the longitudinal velocity, reducing the efficiency of the compensation scheme for a strong focusing lattice.

For the following discussion of the compensation scheme for a strong focusing quadrupole lattice the design parameters of the TESLA X-ray FEL have been used as listed in Table 1. The undulator is built up by an alternating sequence of 0.4 m long quadrupoles and 2.0 m long undulator modules. It is one of the three optional designs for the undulator.

2. Electron motion

The motion within the undulator is split into a fast oscillation, driving the FEL amplification, and a betatron oscillation. For a highly relativistic electron the longitudinal velocity is

$$v_z = c\left[1 - \frac{1+K^2}{2\gamma^2} - \frac{p_x^2 + p_y^2}{2(\gamma mc)^2}\right] \qquad (1)$$

where γ is the beam energy normalized to the electron rest mass mc^2, K is the dimensionless rms undulator field and $p_{x,y}$ are the transverse canonical momenta. For natural undulator focusing, the last term of Eq. (1) is added up with the transverse dependency of the undulator field $K(x,y) = K_0(1 + k_x^2 x^2/2 + k_y^2 y^2/2)$ to a constant term when averaged over one undulator period λ_U, where $K_0 = eB/mck_U$ is the field at the undulator axis. The explicit values of k_x and k_y are defined by the curvature of the magnet pole faces, fulfilling the constraint $k_x^2 + k_y^2 = k_U^2$ with $k_U = 2\pi/\lambda_U$. This constant reduction of the longitudinal velocity makes the compensation by a larger energy for large betatron amplitudes highly efficient. For a strong focusing lattice the longitudinal velocity is not constant anymore. The variation in p_x and p_y can only be compensated in average by a conditioned beam.

The spread in v_z due to the transverse emittance has to be smaller than the acceptance of the FEL amplification resulting in the constraint

$$\varepsilon_N \ll \frac{4\rho\beta\gamma\lambda}{\lambda_U} \qquad (2)$$

where ε_N is the normalized emittance, ρ is the FEL parameter [9,10], β the beta function and λ the radiation wavelength of the FEL. The right-hand side of Eq. (2) drops approximately as γ^{-1} for higher beam energies. The constraint for a small emittance is more severe for an X-ray FEL than for an FEL, which radiates in the visible or IR regime.

To estimate the trajectory of the electrons in a strong focusing lattice and thus the impact of p_x^2 and p_y^2 in Eq. (1), the TESLA undulator quadrupoles are described in the thin lens approximation.

The electron motion in the x-direction for the drift section behind a focusing quadrupole of the periodic FODO lattice is given by

$$x(z) = \sqrt{I_x \beta(z)} \sin(\Psi(z) + \phi_0) \qquad (3)$$

with

$$\beta(z) = \beta_0 - 2\alpha_0 z + \gamma_0 z^2 \tag{4}$$

and

$$\Psi(z) = \arctan(\gamma_0 z - \alpha_0) + \arctan(\alpha_0), \tag{5}$$

where α, β and γ are the optical functions, ϕ_0 the initial betatron phase, I_x the Courant–Snyder constant of the betatron motion [11]. The index 'O' refers to the values of the optical functions right at the beginning of the drift with the length L. The solution for the second drift after the defocusing quadrupole is obtained by replacing z with $L - z$ in Eqs. (3) and (4) and by adding the phase advance $\Psi(L)$ to ϕ_0.

The divergence $x' = p_x/p_z$ is independent of z for the drift section with

$$x'(z) = \sqrt{\frac{I_x}{\beta_0}}[\cos(\phi_0) - \alpha_0 \sin(\phi_0)] \tag{6}$$

for $z < L$ and

$$x'(z) = \sqrt{\frac{I_x}{\beta_0}}\left[\left(1 + \frac{2\alpha_1 L}{\beta_1}\right)\cos(\phi_0) + \left(1 - \frac{2L}{\alpha_1 \beta_1}\right)\alpha_0 \sin(\phi_0)\right] \tag{7}$$

for $L < z < 2L$ with $\beta_1 = \beta(L)$ and $\alpha_1 = \alpha_0 - \gamma_0 L$.

These general results can be simplified if the FODO cell length is small compared to the average value of the beta function $\bar{\beta} = (\beta_0 + \beta_1)/2$ ($L = 2.4$ m, $\bar{\beta} \approx 18$ m for the TESLA FEL).

The values of α_0 and α_1 tend to be unity with

$$\alpha_{0,1} \approx 1 \pm \frac{L^2}{2\bar{\beta}^2}. \tag{8}$$

The upper and lower sign denotes the value of α_0 and α_1, respectively. The transverse divergence consists of two terms of equal strength, where the sine term with an alternating sign over one FODO cell length modifies the electron motion to a 'saw-tooth'-like trajectory at maximum displacement of the electron from the undulator axis (Fig. 1).

With a value of α close to unity and a short FODO cell length the longitudinal velocity exhibits a dominant fluctuation of p_x^2 in Eq. (1) on the scale of $2L$, which is shorter than the gain length. For the FEL process this term can be regarded as constant by averaging over one FODO cell length. The long-term variation with the periodicity of the betatron oscillation is strongly inhibited with a remaining amplitude of a few percent compared to the constant term. Therefore, the compensation scheme becomes efficient for this kind of quadrupole lattice. The applied correlation between energy and the betatron amplitude is given by

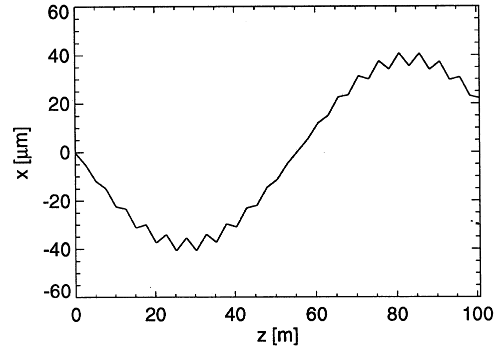


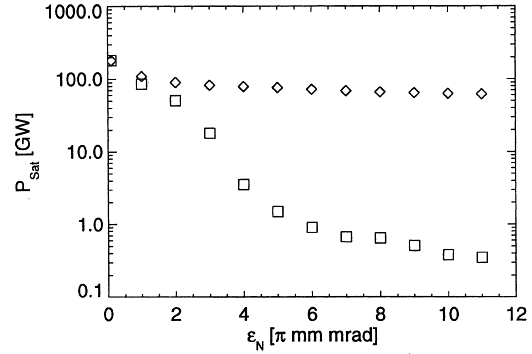
Fig. 1. Trajectory of a sample electron within the undulator of the TESLA X-ray FEL.

$$\Delta\gamma = \frac{\lambda_U \gamma}{4\lambda\bar{\beta}}(I_x + I_y). \tag{9}$$

For a typical quadrupole lattice of an X-ray FEL the impact of the natural undulator focusing is negligible and the correlation strength is the same for both planes.

3. Simulation results

The 3D FEL code GENESIS 1.3 [12] has been used to study the efficiency of the compensation scheme for the strong focusing lattice of the TESLA X-ray FEL. The results for different values of the normalized emittance are shown in Fig. 2. For the largest emittance of $\varepsilon_N = 11\pi$ mm mrad and an unconditioned beam the degradation of the saturation power would be two orders of magnitude while the saturation length is increased by a factor of six.



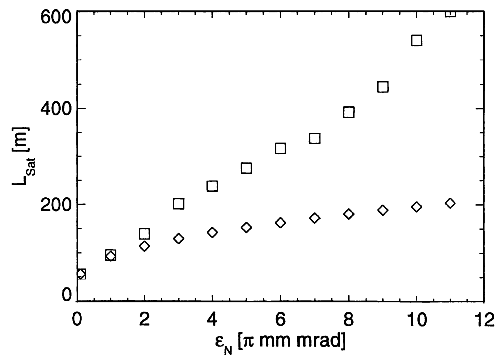


Fig. 2. Saturation power and length (top and bottom, respectively) for different normalized emittances for a conditioned and unconditioned electron beam ((◇) and (□) marker, respectively).

The case of a conditioned beam yields a significantly improved output although the results are still worse than those for the design value of $\varepsilon_N = 1\pi$ mm mrad. This is explained by the fact that a larger emittance increases the electron beam spot size and thus degrades the efficiency of the FEL. Compared to simulations where the focusing strength has been reduced to obtain the same spot size as with the design value of the emittance, the differences in saturation power and length are less than 10%. This indicates that the compensation scheme is highly efficient.

In order to find the highest efficiency, the correlation strength has been varied. The results are presented in Fig. 3. The best performance is achieved for the correlation strength according to Eq. (9). Any different value would reduce the output power as well as increase the saturation length. The results are asymmetric in the correlation strength where an overconditioned beam provides a better performance than an underconditioned one. This fact is explained by the distribution in the longitudinal phase space. An overconditioned beam corresponds to an equivalent energy distribution with a tail towards higher energies. The core process of the FEL amplification can be regarded as a rotation of the distribution in the longitudinal phase space. The tail is shifted to lower energies by transferring energy to the radiation field. This supports the FEL process in contrast to the underconditioned case, where the tail extracts energy from the radiation field and thus weakens the FEL amplification.

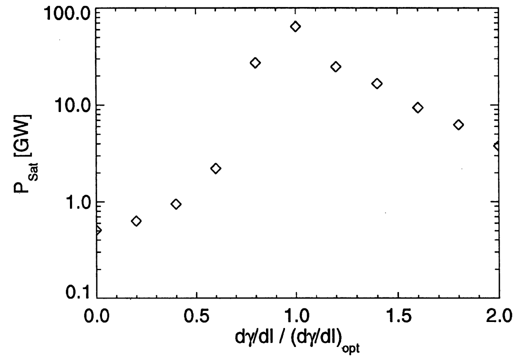


Fig. 3. Saturation power for a conditioned electron beam with different strengths of correlation.

4. Conclusion

Due to the properties of a strong focusing lattice with a FODO cell length several times shorter than the average beta function, the variation of the longitudinal velocity has a characteristic scale shorter than the gain length and can be regarded as constant for the FEL amplification. This makes it suitable for applying a correlation between the betatron amplitude of the electron and its energy to compensate for the reduction in synchronization of the electrons with the radiation field. It relaxes the constraint of a small transverse emittance. This is particularly so for beam energies driving an X-ray FEL.

Acknowledgements

The author would like to thank Y. Derbenev, P.N. Lebedev and J. Rossbach for useful discussions.

References

[1] J. Rossbach et al., Nucl. Instr. and Meth. A 375 (1996) 269.
[2] R. Tatchyn et al., Nucl. Instr. and Meth. A 375 (1996) 274.
[3] Linear Collider Conceptual Design Report, DESY print 97-48, Hamburg, DESY, 1997.
[4] S.V. Milton et al., Nucl. Instr. and Meth. A 407 (1998) 210.
[5] J. Rossbach et al., DESY print TESLA-FEL 95-06, Hamburg, DESY, 1995.
[6] Y. Derbenev, RF compensation of aberrational velocity spread of electrons in FEL, Reported at Light Source Department Seminar, BNL, 1990.
[7] A.N. Sessler, D.H. Whittum, L.H. Yu, Phys. Rev. Lett. 68 (1992) 309.
[8] E.T. Scharlemann, J. Appl. Phys. 58 (1985) 2154.
[9] R. Bonifacio, C. Pellegrini, L.M. Narducci, Opt. Commun. 50 (1984) 373.
[10] D.A. Edwards, M.J. Syphers, An Introduction to the Physics of High Energy Accelerators, Wiley, New York, 1993.
[11] E.D. Courant, H.S. Snyder, Ann. Phys. 3 (1958) 1.
[12] S. Reiche, Nucl. Instr. and Meth. A 429 (1999) 243.