Free-Electron Simulations at Short Wavelengths

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Outline

- FEL Simulations
  - Overview
  - Codes
  - Shot-Noise Models
- Short Wavelength Effects
  - Spontaneous Radiation
  - Wake Fields
  - Undulator Description
- Start-End Simulations
The FEL framework is based on two assumptions / approximations to simplify the equations to be solved.

- **Resonance Approximation**: The interaction between electron beam and radiation field per undulator period is small, thus it requires a resonant behavior at specific wavelengths.

- **Paraxial Approximation**: The radiation field propagates along with the electron beam. Changes in the field amplitude, phase and directions occurs on a slower time-scale than the dominant oscillation at the resonant wavelength.
The FEL Equations

**Electron Energy**

\[
\frac{d\gamma}{dz} = a_1(Ae^{i\theta} + \text{c.c.}) + a_2E_z + W_z
\]

- Radiation field
- Space charge field
- External effects

**Electron Phase**

\[
\frac{d\theta}{dz} = a_3\left(1 - \frac{\gamma^2}{\gamma_R^2}\right) + a_4(Ae^{i\theta} - \text{c.c.}) + a_5p_{\perp}^2
\]

- Resonance condition
- Radiation field
- Betatron motion

**Transverse Motion**

\[
\frac{dr_{\perp}}{dz} = \frac{p_{\perp}}{\gamma mc^2} \quad \frac{dp_{\perp}}{dz} = f(r_{\perp}, \gamma)
\]

- Undulator lattice

**Radiation Field**

\[
\nabla_{\perp}^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\partial t}\right)A = a_6\rho(\theta, r_{\perp})e^{-i\theta}
\]

- Diffraction
- Slippage
- Source term
Analytical & Numerical Models

Collective Beam Parameters
- 1D gain length

Macro Particles
- Saturation

2D/3D Description
- Diffraction, emittance

Time-dependence
- Slippage, shot noise

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## FEL Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Beam</th>
<th>Dimension</th>
<th>Time</th>
<th>Harmonics</th>
</tr>
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<tbody>
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<td>No</td>
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<tr>
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</tr>
<tr>
<td>Nutmeg</td>
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</tr>
<tr>
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<tr>
<td>Sarah</td>
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<td>1D</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>TDA3D</td>
<td>Particles</td>
<td>3D</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
Electron beam is modeled by macro particles.

- ‘Standard’ tracking methods for transverse variables.
- Robust solver (e.g. Runge-Kutta) for energy and ponderomotive phase is preferred to handle discontinuities at undulator module entrance and exit.

Reduction of the radiation field information

- Discretization on a grid (finite-difference, finite-element)
- Decomposition into orthonormal modes (finite-mode)
- Fourier decomposition (finite frequency)

All discretization methods reduce Maxwell’s equation in a (sometimes trivial) matrix equation. The nature of partial differentiation allows numerical instability of the system to be solved.

Alternative methods such as Green’s function or Lienard-Wiechert potentials become inefficient at short wavelengths and long undulators.
Time-Dependent Simulations

- Along bunch
- Along undulator
- Radiation Slice
- Electron Slice
- Periodic B.C.
- Unphysical

CPU time = \# integration steps \cdot \# (electrons + grid points)/slice \cdot bunch length / slice separation / wavelength

Memory = \# integration steps \cdot \# grid points/slice / slice separation
Memory = CPU time for periodic boundary condition
Scaling (rough estimate):

- wavelength ~ $\gamma^2$
- # integration steps ~ $\gamma$ (resolves focusing structure)
- # electrons / slice ~ $\log(\gamma)$ (numerical stability)
- # grid points / slice ~ $\log(\gamma)$ (frequency resolution)
- bunch length ~ $\text{const.}$
- slice separation ~ $\log(\gamma)$ (frequency resolution)

Example:
LCLS @ 1.5 Å, complete bunch, starting from noise
120 Mill. macro particles, 150 Mill. grid points in Genesis 1.3
6 days on a P4, 1.7 GHz, 50 MByte needed memory

Demand for Parallel Computation
Shot Noise

Shorter wavelength reduces diffraction effects and, thus, the formation of transverse coherence (local interaction in transverse plane).

**Paradigm for generating shot noise**

\[
\left\langle |e^{i\theta}|^2 \right\rangle_v = \frac{1}{\int_{u} \rho \, dV} = \frac{1}{n_v}
\]

Macro particles are grouped into beamlets. Every macro particle in beamlet has identical values for transverse variables and energy. The ponderomotive phase is filled using quiet loading algorithm plus:

- Penman algorithm of random offsets
- Construction by modifying a pair with opposite phase + rotation
- Random offsets in Fourier spectrum (higher harmonics)

\[1. + 3.\quad 2.\]
Spontaneous Radiation

The mean energy loss due to spontaneous radiation can be compensated by tapering the undulator field. Taper is typically stepwise (constant over a single undulator module).

Quantum fluctuation in the emission of hard x-ray photons increases the energy spread. This impose a limit of the max. beam energy, driving an FEL.
Undulator Wakefields

Sources of undulator wakefields:
- Resistive walls
- Surface roughness
- Change in aperture

Wakefields have a ‘local’ effect, which cannot be compensated by tapering. A larger gap reduces the wakefield amplitude but increases the saturation length and reduces the saturation power.
For optimum performance the focusing of the undulator lattice should balance out the benefits of a higher electron density with the degrading emittance effects.

The natural focusing of the undulator field cannot provide the required strength for optimum focusing. Strong focusing is added.

Typical undulator lattice
Perturbation of the FEL amplification by

- Diffraction of the radiation field within the module gaps
- Mode-coupling at module entrance
- Phase slippage between electron beam and radiation field
- Debunching for doublet and triplet set-up
- Accumulated ‘transition’ effects
Due to the short coherence/slippage length at short wavelength the FEL process is a ‘local’ effect, where many part of the bunch amplifies the radiation independently. It cannot be assumed that the beam parameters are constant along the beam due to reasons such as:

- Space-charge effects in the electron beam production.
- Wakefields in the linacs and the undulator.
- CSR induced emittance growth and microbunch instability in bunch compressors.
- Effect of RF-curvature and $T_{566}$ in bunch compressors.

Although the performance for complex bunch structures can be estimated by a look-up table of sample runs, covering all possible combination of beam parameters, the FEL simulation code should support arbitrary profiles. The input for these codes comes from other codes, transporting the beam from its creation to the beginning of the undulator.
Start-End Simulations

Module | Primary Problems | Codes
-------|-----------------|---------
Gun     | Space-charge, wide range of electron energy, explicit E- and B-fields | Astra, Parmela
Linac   | Various beam line elements, wakefields | Elegant
Bunch Compressor | CSR | Traffic4
FEL     | SASE, wakefields | Genesis 1.3, Ginger
X-ray optics | X-ray beam transport, monochromator | |

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Distributions cannot be imported directly into FEL codes, because FEL codes resolve a length scale much shorter than those for the preceding tracking codes.

**Methode 1:** Distribution is sliced and the moments (mean and rms values) are calculated. The FEL code imports the sliced beam parameters and ‘recreates’ the distribution.

**Methode 2:** Distribution is sliced and all values except for the long. position are used for the beamlets. The ponderomotive phase is generated in the usual way. If necessary particles are added to or removed from the sliced distribution.

<table>
<thead>
<tr>
<th>Methode</th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Easy to implement</td>
<td>Exotic correlations lost</td>
</tr>
<tr>
<td>2</td>
<td>Conserves distribution</td>
<td>Time consuming, numerical difficult</td>
</tr>
</tbody>
</table>
Saturation has been observed after a change in the beam optics. Running off-crest in the linac the 20° dispersive section acts as a bunch compressor. In addition an aperture limit causes parts of the bunch to be scraped, depending on the linac phase.
## VISA: Electron Beam Properties

### Beam Parameters at Undulator Entrance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>71 MeV</td>
</tr>
<tr>
<td>Spread</td>
<td>0.1 %</td>
</tr>
<tr>
<td>Peak Current</td>
<td>250 A</td>
</tr>
<tr>
<td>Emittance (projected)</td>
<td>2.3 mm·mrad</td>
</tr>
<tr>
<td>Undulator period</td>
<td>1.8 cm</td>
</tr>
<tr>
<td>Undulator parameter</td>
<td>0.88</td>
</tr>
<tr>
<td>Wavelength</td>
<td>850 nm</td>
</tr>
</tbody>
</table>

### Bunch Length at Undulator

![Graph of CTR Signal vs. Relative Linac RF-phase](image1)

### Current Profile (Simulation)

![Graph of Current vs. Location](image2)

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Simulation shows good agreement with experiment:
- Power growth
- Spectrum + bandwidth
- Near & far field distribution
- (Fluctuation - not enough runs)
Nominal *LCLS* Linac Parameters for 1.5-Å FEL

- **7 MeV**
  - \( \sigma_z \approx 0.83 \text{ mm} \)
  - \( \sigma_\delta \approx 0.2 \% \)
  - \( \tilde{\sigma}_z \approx 0.6 \%

- **150 MeV**
  - \( \sigma_z \approx 0.83 \text{ mm} \)
  - \( \sigma_\delta \approx 0.10 \% \)
  - \( \tilde{\sigma}_z \approx 0.6 \%

- **250 MeV**
  - \( \sigma_z \approx 0.19 \text{ mm} \)
  - \( \sigma_\delta \approx 1.8 \% \)
  - \( \tilde{\sigma}_z \approx 1.8 \%

- **4.54 GeV**
  - \( \sigma_z \approx 0.022 \text{ mm} \)
  - \( \sigma_\delta \approx 0.76 \% \)
  - \( \tilde{\sigma}_z \approx 0.76 \%

- **14.35 GeV**
  - \( \sigma_z \approx 0.022 \text{ mm} \)
  - \( \sigma_\delta \approx 0.02 \% \)
  - \( \tilde{\sigma}_z \approx 0.02 \%

**Linac-0**
- \( L = 6 \text{ m} \)
- \( \varphi_{rf} = 0 \)
- **Linac-1**
- \( L = 9 \text{ m} \)
- \( \varphi_{rf} = -38 \degree \)
- **Linac-2**
- \( L = 330 \text{ m} \)
- \( \varphi_{rf} = -43 \degree \)
- **Linac-3**
- \( L = 550 \text{ m} \)
- \( \varphi_{rf} = -10 \degree \)

**Triplet**
- \( R_{56} \approx 0 \)
- **BC-1**
- \( L = 6 \text{ m} \)
- \( R_{56} = -36 \text{ mm} \)
- **BC-2**
- \( L = 22 \text{ m} \)
- \( R_{56} = -22 \text{ mm} \)
- **undulator**
- \( L = 120 \text{ m} \)

**Existing Linac**
- **SLAC Linac Tunnel**
- **FFT/B Hall**
LCLS: Beam Transport

Parmela - Elegant Output at Undulator Entrance

- $\alpha, \beta$-mismatch
- 4D centroid osc. amplitude
- Analytical formulae
LCLS: FEL Performance

- Start-end simulations for two different initial bunch charges of 1 nC and 0.2 nC.
- Low charge case is modeled after the GTF results and then propagated through the LCLS beam line.
- The impact of wakefields is a reduction of the output power by 35%.

Transverse coherence is reached after 35 m. The low charge case performs worse than the high charge case. The reason is the large slice emittance, while the current is smaller by a factor of 3 (The high charge case assumes an ideal performance at the gun).
Head and tail region strongly suppressed due to large wake amplitude and poor beam quality. Even without wakefields the ‘gaps’ in the profile are present. High charge case has a shift towards longer wavelength due to the higher net losses by wakefields.
Conclusion

- Numerical codes extend the framework of the analytical FEL model.
- Various codes exist, benchmarked against each other and experiments.
- Undulator lattice, wakefields and spontaneous radiation become important at shorter wavelengths.
- Electron beam generation and transport included in the simulations (start-end simulations)
- Start-end simulations successfully applied to the VISA-FEL, LEUTL-FEL and TTF-FEL results. Extrapolation to shorter wavelength (TESLA-FEL and LCLS)