A Fast Method to Estimate the Gain of the Microbunch Instability in a Bunch Compressor

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To reach high peak currents driving Free-Electron Lasers an initial chirped electron bunch is compressed in a bunch compressor. The interaction of the electron beam with its radiation field can yield a collective instability, which amplifies any initial modulation in the current profile. We present a model, which allows to derive an explicit analytical expression for the gain of the microbunch instability. The results are compared to those of the more complex analytical models.

1. Introduction

Many FEL experiments [1] require a bunch compressor to increase the peak current. The magnetic chicane resembles a single period of an undulator and like an FEL [2,3] the interaction between the coherent synchrotron radiation (CSR) [4] and the electron beam can enhance the amplitude of an initial current modulation [5]. The mechanism has striking similarity to the FEL process, although the electron motion is more complex than the averaged motion needed for the FEL model. The interaction with the CSR field is expressed by a potential.

2. Motion in the Magnetic Chicane

In our model we consider a magnetic chicane with a bending radius $R$, length $L$ for the outer magnets and $2L$ for the inner one and no drift spaces. We integrate over all $R_{56}$-matrix element between the chicane entrance and the current position $s$. The longitudinal velocity becomes for the small bending angle approximation ($L \ll R$)

$$\frac{d\zeta}{ds} = \int_0^s \frac{\delta(s') \cdot s - s'}{R(s)R(s')} ds'$$

(1)

where $\zeta$ is the particle position in the co-moving frame and $\delta = (\gamma - \gamma_0/\gamma_0)$ is the normalized deviation from the mean beam energy $\gamma_0$. Note that any change in the particle energy evolves at least in third order in $s$. Further, the change of polarity at the second dipole decreases part of the accumulated velocity.

The energy modulation, $\delta$, is induced by the potential of the CSR radiation [4]. Assuming a coasting beam with a relative current modulation $b$ the electron energy changes as

$$\frac{d\delta}{ds} = -\frac{I_0}{I_A \gamma_0} \frac{2\Gamma \left(\frac{4}{3}\right) k^+}{2R^2} |b(s)| \sin \left(k\zeta + \frac{\pi}{3}\right)$$

(2)

where $I_0$ is the beam current, $I_A$ is the Alven current and $k$ is the wavenumber of the modulation.

3. Low Gain Model

We assume in our low gain model that a change in the current modulation occurs mainly in the second half of the chicane, driven by the accumulated energy change over the first half. Each electron has a constant energy change. The amplitude and sign depends on its position within the CSR potential. By holding $b(s)$ constant we integrate over Eq. 2 and then Eq. 1. We define the normalized, longitudinal amplitude as

$$\xi = \frac{I_0 \Gamma(2/3)}{2I_A \gamma_0} \left(\frac{8L^3 k}{3R^2}\right)^{1/2} \Phi(s/L)$$

(3)

with

$$\Phi(x) = \begin{cases} \frac{1}{64} x^4 & x \leq 1 \\ \frac{1}{64} x^4 - \frac{3}{16} x^2 + \frac{1}{2} x - \frac{1}{16} & 1 < x \leq 3 \\ \frac{1}{64} x^4 - \frac{2}{16} x^2 + \frac{1}{2} x - \frac{7}{16} & 3 < x. \end{cases}$$

(4)

The gain of the microbunch instability is defined as the ratio between final and initial modula-
tion amplitude and is
\[ G = e^{-\alpha |\xi|^2} \sqrt{1 + |\xi|^2}, \]  
(5)
with the normalized energy spread \( \alpha = (I_A\gamma_0 / 2I_0 \Gamma(2/3))^{2/3}\sigma_3^2 \) and \( \sigma_3 \) as the rms size of the initial distribution in \( \delta \).

For LCLS like parameters (\( \gamma_0 = 500, I_0 = 100A, R = 12m, L = 1.5m \) and \( k = 1.25 \cdot 10^6 m^{-1} \)) the gain would be 25. The comparison with a complete self-consistent theory [5] is shown in Fig. 1. Despite the simplicity of our model it agrees within a few % with the exact results.

4. High Gain Model

If the motion within a single magnet enhances the current modulation so that the emission of the coherent synchrotron radiation is stimulated, the low gain model is no longer applicable, because it assumes a negligible grow in the modulation. To obtain a quantitative expression for this limit we solve Eqs. 1 and 2 self-consistently by introducing the collective variables \( B = -ik < e^{-i\Psi} \xi > \) and \( \Delta = < e^{-i\Psi} \delta > \), where \( \Psi_j = k\zeta_0,j \) is the initial equidistant distribution of the electrons. The equations of motion for a cold beam (no energy spread) becomes a 4th order differential equation
\[ \frac{dB}{ds} = \frac{\rho_{csr}}{R^4} e^{i\Delta B} \]  
(6)
with the dimensionless parameter
\[ \rho_{csr} = \left[ \frac{I_0}{I_A\gamma_0} \right]^{1/3} \Gamma \left( \frac{2}{3} \right)^{4/3} (kR)^{1/3} \]  
(7)

The ansatz \( B = \exp[i\Lambda s] \) yields a 4th order dispersion equation. Two roots have negative imaginary parts corresponding to an exponentially growth of the instability with a growth rate of \( \rho_{csr}/R\sin(7\pi/24) \) and \( \rho_{csr}/R\sin(5\pi/24) \). The condition for a significant growth over a single dipole is \( \rho_{csr} > 4R/L \), where the factor 4 arises from the mode competition in the start-up regime. Although not included in our model energy spread suppresses the collective instability. An estimate for a Gaussian energy spread yields the maximum energy spread at which amplification occurs as \( \sigma_3 < \rho_{csr}^3/10kR \).

Figure 1. gain along magnetic chicane for low gain model and self-consistent solution (solid and dashed line, respectively).

5. Conclusion

We derived a simple model to quickly estimate the gain of the microbunch instability within the bunch compressor. This model is valid as long as the collective instability does not operate in the high gain regime, which validity is given by an effective \( \rho_{csr} \)-parameter much larger than unity. For the example of a generic LCLS chicane this is not the case and the results of the low gain model agrees well with the self-consistent solution.

REFERENCES