A laser-pumped accelerator utilizing a resonant, periodic, dielectric-loaded structure is proposed. The electromagnetic fields due to a side-injected laser beam impinging on this structure are described in a two-dimensional standing-wave approximation, and analyzed in terms of their suitability for accelerating electrons to ultra-relativistic energies. The longitudinal dynamics of injected electrons in the capture phase of the motion are discussed, as are the first and second order transverse focusing effects inherent in this type of structure. Similarities and differences between this scheme, and conventional as well as advanced accelerators, and practical issues relevant to experimental realization of laser acceleration are examined.

1. Introduction

Advanced acceleration techniques[1-7] are currently a very active area of basic research, motivated by a range of applications which spans from high energy physics to radiation generation and medicine. In particular, there have been numerous recent advances in acceleration of electrons by exploiting the large electric fields available in high power laser pulses. As the electric field is naturally transverse in a laser pulse, in order to impart energy gain secularly to an electron, either the electric field must be rotated to have a longitudinal component, as in the laser wake-field accelerator[1], the inverse Smith-Purcell effect[2], and the inverse Cerenkov accelerator[3], or the electron trajectory must have a periodic transverse component, as in the inverse free-electron laser[4]. These schemes have a common drawback, however, in that they demand very high peak laser power to obtain large acceleration gradients.

More promising schemes employ resonant excitation of a structure to build up large field amplitudes. Resonant accelerators utilize smaller peak powers to excite the acceleration fields, and can be efficiently beam-loaded by long pulse trains, leading to higher average output currents and efficiencies. Perhaps the best example of this sort of scheme which uses lasers is the plasma beat-wave accelerator [5], in which the accelerating fields are provided by resonantly exciting electrostatic plasma waves. Unfortunately, because of the presence of free plasma electrons and associated currents inside of the beam channel, the transverse and longitudinal fields in this scheme are in general not of as high quality for accelerating low energy spread and emittance beams as more conventional accelerators[6]. Serious problems in maintaining beam quality are not unique to plasma-based schemes, but are also present in laser accelerators with asymmetric geometries such as gratings[2].

To remove these objections to present laser-based acceleration schemes, we should therefore look for a resonantly excited, symmetric, purely electromagnetic structure. One might imagine that plasma waves could provide such a structure, but since plasmas must be driven nonlinearly to remove all the electrons from the beam channel, they do not naturally form a useful resonant system. This problem could be surmounted in principle by use of a hollow plasma channel, as proposed by Katsouleas, et al.[7], but this scheme raises serious technical issues.
2. Resonant Optical Cavities

A laser-pumped resonant accelerator may be easily constructed, however, using simple periodic metal or dielectric structures. As dielectric systems are attractive from the point of view of lower losses at optical frequencies, we will examine a dielectric-based system in this paper. In the end, when the operating frequency of the accelerator is chosen, the issue of the type of structure to employ will be decided by practical considerations such as damage thresholds and losses.

To begin the conceptual description of the structure, we must first consider the trapping of the laser power in a cavity, which can be thought of as the gap in a Fabry-Perot interferometer made of a pair of highly reflective dielectric or metallic mirrors. For the case of the dielectric mirrors, the field trapped in the gaps can likewise be considered a defect mode of the photonic band gap system[8][9]. In a pulsed system such as we are considering, the power incident on this mirror pair is initially nearly entirely reflected, while in steady state nearly all power is transmitted by the mirror-cavity system, after the cavity fills with energy density to the level where equilibrium is established.

A large stored energy $U$, and therefore large electromagnetic field amplitudes in this cavity system, can be obtained with low external power $P$ by having a large quality factor $Q = \omega U/P$. The $Q$ of the cavity system operating at angular frequency $\omega$ also controls its exponential filling time, $\tau_f = Q/\omega$. In general, it is desirable to have a large $Q$, and filling time, but the laser pulse length may be limited by laser damage effects. Recent studies by Du, et al. of optical damage in SiO$_2$[12] have shown that for very short laser pulses, $\tau_l \leq 1$ psec, the damage threshold of laser energy per unit area $U_l/A$ has been found to scale as $\tau_l^{-1}$, while for longer pulse lengths the threshold the pulse length dependence of the threshold becomes $\tau_l^{1/2}$. This implies that the peak allowable power per unit area is proportional to $\tau_l^{-2}$ for $\tau_l \leq 1$ psec, and $\tau_l^{-1/2}$ for $\tau_l \geq 10$ psec, scalings which strongly favors shorter pulse lengths and thus the choice of $Q$ and operating frequency. It is also important to note that a resonant laser accelerator, while requiring peak power scaling as $Q^{-1}$, would suffer breakdown at the same field level as a nonresonant structure, as it is the field amplitude which dictates the material damage through avalanche ionization. For an example relative to our present discussion, at $\tau_l = 2.5$ psec, the limiting energy flux is approximately 3 J/cm$^2$[12], with an associated peak field amplitude of 2.5 GV/m.

3. Accelerating Field Modulation

To create a useful accelerating electromagnetic field profile, in addition to resonantly storing the power, one must also modulate the field component in the direction of the electron motion. This modulation must have a spatial wave-number which is equal to the free-space wave-number of the laser light $k_0 = \omega/c$, in order to create a spatial harmonic component (a component having a dependence in the direction of electron motion which is some harmonic of the structure periodicity[11]) of the field having phase velocity equal to the speed of light. This spatial harmonic can then effectively accelerate a relativistic ($v = \beta c \approx c$) electron. There are of course many ways of introducing a phase modulation, which include periodic boundaries (e.g. an undulating wall), and period changes in the medium. Here we discuss a variation on the latter scheme, the use of a uniform thickness dielectric mask with a permittivity which is a periodic function of $z$, having a period $d = \lambda_0 = 2\pi/k_0$, the free space wavelength. Furthermore, for the purpose of a model calculation, let us assume the following mathematical form of the permittivity within the mask,
\[ \varepsilon(z) = \varepsilon + \frac{\Delta \varepsilon}{2} \cos(k_0 z), \]  

(1)

where \( \varepsilon = (\varepsilon_{\text{min}} + \varepsilon_{\text{max}})/2 \) and \( \Delta \varepsilon = \varepsilon_{\text{max}} - \varepsilon_{\text{min}} \). The geometry of this system, which is assumed, for the purpose of discussion, infinite in the \( x \) dimension, is shown schematically in Figure 1, with metallic mirrors chosen to allow the outer dielectric boundary condition to be approximated as perfectly conducting. This is a powerful model to begin the analysis, as its solutions can easily be generalized to include any periodically varying permittivity.

![Figure 1. Schematic representation of the top half of one period of a side-injected laser acceleration resonant structure using a modulated dielectric mask, with width in the \( x \) dimension much greater than \( \lambda_0 \), and a metallic mirror.](image)

In this model, we can find the symmetric TM modes of the structure by assuming a separable form of the longitudinal electric field \( E_z = E_0 Y(y)Z(z)e^{i\omega t} \), where we have ignored the \( x \) dependence, assuming it occurs on a scale much larger than a free-space wavelength. The familiar wave solution obtained for the vertical dependence of \( E_z \) is, thus assuming a field null at the mirror-mask boundary;

\[ Y(y) = \sin[k_y (y - (b + a))], \]

(2)

where the vertical wave-number \( k_y \) is a separation constant which remains to be determined. The longitudinal dependence of \( E_z \) within the dielectric is then given by

\[ \frac{d^2 Z}{dz^2} + \left[ k_0^2 \left( \frac{\varepsilon + \Delta \varepsilon}{2} \cos(k_0 z) \right) - k_y^2 \right] Z = 0, \]

(3)

which can be recognized as the Mathieu equation.
Since we are interested in maximizing the first higher spatial harmonic, the solution to the Mathieu equation of most interest has two zeros in a period interval. This solution to Eq. 3 can be written approximately as

\[
Z(z) = \cos(k_0 z) - \Delta \epsilon \left( \frac{\cos(2k_0 z) - 1}{12} - \frac{1}{4} \right) + \Delta \epsilon^2 \left[ \frac{\cos(3k_0 z)}{384} - \frac{19 \cos(k_0 z)}{288} \right] + O(\Delta \epsilon^3), \tag{4}
\]

The amplitude of the component of this standing wave which has phase velocity \(c\) (the forward wave of the \(k_z = k_0\) spatial harmonic) is, relative to all the other spatial harmonics, maximized for \(\Delta \epsilon\) much less than unity. From this point of view, it is best to choose \(\Delta \epsilon\) as small as possible, consistent with the demands of coupling the incoming laser light to the TM accelerating mode, as this minimizes both the field amplitude and power loss associated with useless spatial harmonics, as well as the contrast in the permittivity needed. The details of coupling are beyond the scope of this paper, but a useful, if simplistic, estimate of optimum coupling can be obtained by requiring that the relative modulation of the phase of an incident plane wave due to differing optical path lengths between the regions of highest and lowest permittivity is maximized,

\[
\sqrt{\epsilon_{\text{max}}} - \sqrt{\epsilon_{\text{min}}} k_0 b \approx \pi.
\]

Once the value of \(\Delta \epsilon\) is chosen based on these considerations, limiting the field to two significant spatial harmonics \((k_z = 0, k_0)\), the vertical wave-number is then given by

\[
k_y \equiv k_0 \sqrt{\epsilon - 1 + \frac{5}{48} \Delta \epsilon^2}.
\]

If the thickness of the dielectric mask is also chosen to yield \(k_y b = n\pi/2, n\) odd, then \(\partial E_z / \partial y = 0\) vanishes at the vacuum-dielectric interface. This allows for a smooth transition to the \(k_z = k_0\) spatial harmonic within the gap which, since it corresponds to \(k_y = 0\), is a constant - i.e., the acceleration due to this resonant component is independent of transverse position in the gap. The gap half-width \(a\) is constrained by the relation \(k_0 a = m\pi, m\) integer, a condition arising from requiring mirror symmetry of the variation of the fundamental \((k_z = k_0, k_y = 0\) in the free-space gap region) spatial harmonic, which we have minimized by our choice of mask thickness. Inside of the gap, therefore, the largest component of the field is a \(k_z = k_0\) standing wave whose forward component can resonantly accelerate relativistic electrons with maximum force

\[
\mathcal{F}_z = -eE_z \equiv \sqrt{\frac{4\pi e^2 QI}{c\left[1 + (\Delta \epsilon^2/8)\right]}}, \tag{6}
\]

where \(I\) is the incident laser intensity, the average is over a period of the structure, and we have assumed that the coupling of the laser to the cavity is unity.

For the purpose of example, a list of parameters is given in Table 1 which describe the salient characteristics of a side-injected laser accelerator based on a modulated dielectric. In addition, the accelerating field profiles as a function of \(z\) and \(y\) are shown in Figs. 2(a) and (b), respectively. The shortness of the laser pulse (which at 2.5 psec is chosen to give high breakdown threshold) compared to the electron transit time through the structure (67 psec) introduces an additional problem of requiring a correlation between the transverse and longitudinal laser intensity profile to ensure that the accelerating field is present at the correct time in the structure. This can be accomplished through use of a dielectric "stair-
case” to produce the necessary delay in the arrival time of the laser power at larger values of $z$. Care must be taken in specification of this component, however, as with any component which affects the phase of the incoming light. The maintenance of spatial coherence across the laser wave-fronts is critical to this scheme.

2. (a) Longitudinal dependence at the symmetry plane ($y = 0$) of $E_z$ for parameters given in Table 1. The offset from zero of the oscillation is due to the ($k_z = 0$) fundamental space harmonic. (b) Vertical dependence of $E_z$ for parameters given in Table 1. The modulation of $E_z$ is due to the presence of the fundamental space harmonic.
It should be noted that this type of accelerator is conceptually similar to a drift-tube linac[13], in that the component of the accelerating field which can resonantly interact with a relativistic electron is created by periodically diminishing the zero-mode excitation. Although only the forward wave component of this standing wave can impart net acceleration to high energy electrons, the backward wave component can provide second order transverse focusing for accelerating electrons, as discussed below.

4. Accelerating Electron Dynamics

The longitudinal dynamics of electrons in this type of structure are more interesting than in high gradient microwave accelerators, because the phase slippage is significant even for fairly relativistic electrons, due to the shortness of the wavelength. This slippage can be counteracted at low energies, as one can taper the periodicity length of the dielectric mask in \( z \) to satisfy \( k_x(z) = 2\pi/d(z) = \omega/v(z) \), matching the phase velocity of the accelerating component of the field to the velocity of the electrons. If the field amplitude is also ramped positively in the transrelativistic injection region, then an initially nearly monochromatic, continuous beam of low energy electrons may be adiabatically captured into small regions of phase, allowing a for very short micro-bunches with minimal energy spread. This type of capture and acceleration scheme is in fact typical of proton linacs (such as an RFQ[13], or a drift-tube linac), where the transrelativistic region also has many spatial periodicity lengths.

<table>
<thead>
<tr>
<th>Laser wavelength (( \lambda_0 ))</th>
<th>1.05 ( \mu )m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average permittivity (( \bar{\varepsilon} ))</td>
<td>5.00</td>
</tr>
<tr>
<td>Permittivity contrast (( \Delta \varepsilon ))</td>
<td>1.41</td>
</tr>
<tr>
<td>Dielectric mask thickness (( b ))</td>
<td>1.66 ( \mu )m ( (n = 15) )</td>
</tr>
<tr>
<td>Gap half-height (( a ))</td>
<td>1.575 ( \mu )m ( (m = 3) )</td>
</tr>
<tr>
<td>Laser illumination dimensions (( l_x, l_z ))</td>
<td>20 ( \mu )m, 2 cm</td>
</tr>
<tr>
<td>Quality factor (( Q ))</td>
<td>500</td>
</tr>
<tr>
<td>Fill time (( \tau_f ))</td>
<td>0.28 psec</td>
</tr>
<tr>
<td>Laser intensity (( I ))</td>
<td>8.4 GW / cm(^2)</td>
</tr>
<tr>
<td>Laser pulse length (( \tau_l ))</td>
<td>2.5 psec</td>
</tr>
<tr>
<td>Average acceleration gradient (( e\bar{E}_z ))</td>
<td>1.0 GV/m</td>
</tr>
</tbody>
</table>

Table 1. Example parameters for a high gradient, resonant side-injected laser accelerator.

In addition to longitudinal or phase focusing, the transverse focusing associated with the capture section can also be used to match the beams' vertical phase space. This is accomplished through two mechanisms arising from transverse electromagnetic forces, which can be written for small vertical displacements as \( F_r \cong ey(dE_z/dz) \). Strong second-order transverse focusing has been shown to exist in standing wave accelerators due to the ponderomotive force of the backward wave[14]. In addition, focusing force first order in the field amplitude is present whenever there is a transient increase in this amplitude[15]. Generalizing the analysis of this focusing in Refs. 14 and 15 to include the first order term
and allowing transverse field variations only in $y$, we obtain an equation describing the vertical motion of a relativistic electron,

$$y'' + \left( \frac{\gamma'}{\gamma} \right) y' + \left[ \frac{\gamma''}{\gamma} + \frac{1}{2} \left( \frac{\gamma'}{\gamma} \right)^2 \right] y = 0. \quad (7)$$

Here the prime indicates the derivative with respect to distance along $z$, and we have defined for ultra-relativistic electrons the normalized energy gain averaged over a period of the structure, $\gamma'(z) \equiv -eE_z(z)/m_ec^2$. The term proportional to $\gamma'$ is responsible for adiabatic damping of the motion, and the terms proportional to $\gamma''$ and $(\gamma')^2$ and due to the first and second order (ponderomotive) electromagnetic focusing effects, respectively.

As an example of how these focusing effects may be implemented in this system, we take the parameters listed in Table 1, $eE_z = 1 \text{ GeV/m}$ ultimate average acceleration, which rises over a length 0.5 cm ($\gamma' = \gamma'_{max}/L_r$). We can gain insight into the focal characteristics of the system by noting that the betatron wave-numbers $(k_\beta)$ associated with the first and second order focusing are approximately equal for the case $\gamma = 7$, with the combined focusing function of 0.5 cm. This is in fact a desirable condition, as it has recently been shown in the context of plasma focusing of electron beams[16] that when $\beta_f = L_r$ one can most straightforwardly obtain a match of the beam's transverse phase space to the focusing channel. A matched beam envelope is indeed possible under conditions of constant acceleration, as noted in Ref. 14, with rms beam size $\sigma_{y,eq} = (2)^{1/4} \sqrt[4]{\varepsilon_{y,n}/\gamma'}$, where $\varepsilon_{y,n}$ is the normalized rms vertical emittance[14][17]. To further illustrate this point, the envelope equation corresponding to the above example, with $\gamma_0 = 2$, $\gamma_f = 7$, and $\varepsilon_{y,n} = 10^{-10}$ m-rad, is integrated, with the results shown in Figure 3.

3. Evolution of rms envelope (solid line) in a linearly rising field region ($L_r = 5 \text{ mm}$) with all other parameters as given in Table 1. The dashed line gives envelope in the absence of the focusing and accelerating electromagnetic field.
As can be seen from this example, it is the electromagnetic focusing which allows the propagation of the beam through the gap of the structure. We must emphasize that the physical basis of this effect is beneficial only symmetric structures. In asymmetric systems like the inverse Smith-Purcell accelerator (which have been proposed due to ease of coupling the laser to the structure), transverse electromagnetic forces will produce uncompensated deflections due to ponderomotive effects, not focusing. It is also interesting to note that the transverse emittance required for this scheme is very small, many orders of magnitude below those injected into rf electron linacs. This is a general feature of acceleration at optical wavelengths, as the beam size must scale with the wavelength to pass through any acceleration structure. Likewise, the longitudinal emittance of an electron bunch captured and accelerated in an optical wave will be much smaller than what is achieved in microwave devices. One can view this scaling both as a technical challenge, and as a desirable result of laser acceleration, since beams of such small phase space volume would undoubtedly lead to many advances in electron beam-based sciences.

In addition to independent particle dynamics, one must be concerned with collective effects, most importantly the space-charge fields of the beam which are deleterious at low energy, and also the beam-excited transverse modes in an accelerator, which can lead to an instability known commonly as beam break-up[18] (BBU). The geometry of the beam, which is much larger in the $x$ than the $y$-dimension, by itself will mitigate the space-charge effects in comparison to a cylindrically symmetric beam[14]. Also, it is well known that short wavelength structures electromagnetically couple more strongly to the beam current due to the proximity of the walls of the structure to the beam axis. Since the structure under consideration is open horizontally, and will only confine well the mode being externally pumped[8], any beam excited transverse modes should radiate away, and not feed back on the beam dynamics. Thus this type of structure should have a high threshold current for BBU, which is important for allowing high levels of beam-loading, and thus high power efficiency.

5. Conclusions

The promise of the system we have described in this paper is to obtain a compact, laser excited, high gradient, inexpensive electron accelerator with attractive injection and transport properties. Most of the demands on the laser system, which in the example uses only 200 $\mu$J per pulse, can be met with today's commercial systems, with some issues requiring additional attention, such as temporal flattening of the laser pulse to provide a uniform accelerating field over most of the laser irradiation time. In addition, the example we have provided here uses 1$\mu$m light (typical of that produced by a Nd:YLF laser), while it is clear that initial proof-of-principle experiments will be easier if performed with longer wavelengths, easing the structure manufacture and alignment, as well as the injected electron source requirements, which are beyond the state of the art in our example. However, there is much reason to believe that the dielectric structures we have described are not overly challenging to build; nanofabrication techniques developed in recent years, for advanced applications such as gradient index optics[19], and microcavity lasers[20], can also meet most of the requirements for construction of the dielectric masks.

The experimental development of this type of accelerator should be within reach of the advanced accelerator community, as the concepts underlying it are nearly identical to known linear accelerators. The innovation here lies entirely in the desire to use inexpensive optical radiation, which requires a rethinking of the accelerator structure, and coupling system design to meet the demands of beam acceleration and transport as they are scaled to shorter wavelengths.
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