Design Considerations for the UCLA PBPL
Slit-based Phase Space Measurement Systems

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The phase space measurement system initially implemented by Spencer Hartman on the UCLA PBPL rf photocathode gun has been upgraded, and a new system has been designed to measure the emittance at higher energy, after the emittance compensating drift and acceleration in the PWT linac. The purpose of this note is to describe the design criteria and physical principles involved in obtaining systems which provide the resolution in phase space measurements that we require. The final slit and detector hardware designs are included; the video data acquisition and analysis will remain nearly unchanged from Hartman's system.

The purpose of collimating the high intensity electron beam with the slits in these devices is two-fold. The first purpose is of course to separate the beam into many beamlets, whose intensity distribution at some downstream point can be measured to give the phase space distribution of the beam; the width of each beamlet gives a measure of the width of the transverse momentum distribution at each slit, and the centroid of the beamlets gives the correlated offset of the momentum distribution at each slit.

The second purpose is of prime importance for our beam, since it is space-charge dominated for almost all energies and beam-sizes of interest after the gun. This is quantified by comparing the space charge and emittance terms in the rms beam envelope equation in a drift space

\[ \sigma'' = \frac{\varepsilon_n^2}{\gamma^2 \sigma_x^3} + \frac{4I}{\gamma^3 I_0 (\sigma_x + \sigma_y)} \]

where \( I \) is the peak beam current, \( I_0 = ec/r_e \) is the Alfven current, the beam is assumed relativistic (\( \gamma >> 1, \beta \approx 1 \)), \( \varepsilon_n \) is the normalized emittance, and, of course, an analogous equation exists for \( \sigma_y \). Now, taking the ratio of the second to the first terms on the right hand side of the envelope equation, and assuming a round beam (\( \sigma_x = \sigma_y = \sigma_0 \)), we have a measure of the degree of space charge dominance over emittance in driving the evolution of the beam envelope,
One can see that for the relevant energies in our beamlines and experiments, that our high brightness beams are space-charge dominated \((R_0 \gg 1)\) except near small waists, and thus linear transport theory cannot be used to measure the emittance \((e.g.\, \text{quad scanning})\). Collimation with slits mitigates this situation, however, by creating low current, small \(\sigma_x\) beamlets which have the same uncorrelated "temperature" as the original beam. Noting that the rms size of a uniform beamlet created by a vertical slit of width \(d\) is \(\sigma_x = d / \sqrt{12}\), and assuming \(\sigma_x \gg \sigma_y\), we have a space-charge dominance ratio for the beamlets,

\[
R_b = \frac{2I\sigma_y^2}{I_0 \gamma \epsilon_n}.
\]

This is ratio is similar in slit size scaling to that found by Hartman for round holes. For our case, at low energy \((E = 4 \, \text{MeV}, \; I = 200 \, \text{A}, \; \epsilon_n = 4 \, \text{mm-mrad})\), it is acceptable to have \(d = 50 \, \mu m\) slits (see the calculation sheet included as an Appendix), that is \(R_b \ll 1\), and the beamlets are emittance dominated. It is apparent that, at least from this point of view, that at high energy the same slit width will be adequate, as the ratio scales as \(\gamma^{-1}\).

There are, in fact, other design considerations which impact on the choice of slit width, having to do with the angular acceptance of the slits. The depth of the material (stainless steel) used to intercept the beam is dictated by our desire to either stop the beam or scatter it sufficiently so that it doesn't affect the measurement of the nonintercepted beamlets. The stopping distance of the beam is approximately

\[
L_s = \frac{E}{\frac{dE}{dx}} \approx \frac{E(\text{MeV})}{1.5(\text{MeV-cm}^2/\text{g}) \rho(g-\text{cm}^{-3})}
\]

for an initially minimum ionizing particle. It is straightforward to stop a 4 MeV beam, but with a 16 MeV beam, the length of the slits becomes impractical. The beam scatters off of nuclei as it as it slows down from ionization losses, and the final rms angle associated with the beam after propagating a distance \(L\) in the stopping material is approximately
\[
\theta_{sc} = \frac{21}{E(\text{MeV})} \sqrt{\frac{L_s}{L_r \left( \frac{1}{L_r} - 1 \right)}},
\]

where \( L_r \) is the radiation length in the material (1.4 cm in steel). If we require a multiple scatter angle of approximately unity, then a 16 MeV beam will need about 5 mm of steel to intercept and scatter it; this is in fact the design value we have chosen.

Once this length has been chosen, one can examine the angular acceptance of the slits. The first thing one needs to do is to specify an rms beam angle associated with the finite beam emittance, which assuming we place the slits at a waist, is

\[
\phi = \frac{\varepsilon_e}{\gamma \sigma_0}.
\]

This angle, which for an beam size of \( \sigma_0 = 1.5 \) mm, with \( E = 4 \) MeV, \( \varepsilon_e = 4 \) mm-mrad, is \( \phi = 0.5 \) mrad. This should be much smaller than the half angle of the slit aperture, as it is for our case, in which \( d / 2L = 5 \) mrad.

The slit separation \( w \) is chosen to be much larger than the slit width \( d \) and smaller than the beam size, to ensure that we can resolve the beam. In our case, the slit width is taken to be 0.75 mm. This width must also be consistent with not allowing the beamlets to overlap at the detecting phosphor, a condition which depends on the distance of the drift to the phosphor \( L_d \). The ratio to the beamlet widths to their separation, which should be much smaller than unity, is

\[
R_{ws} = \frac{2L_d \phi}{w},
\]

while the ratio of the beamlet rms size at the phosphor to its size at the slit, which should be larger than one to achieve resolution of the uncorrelated angular spread in the beam, is

\[
R_{sp} = \sqrt{\frac{L_d \phi}{d}}.
\]

Since one of these ratios should be small and the other large compared to unity, if we set their geometric average equal to unity \( (R_{sp} R_{ws} = 1) \), we can optimize the drift length to be

\[
L_d = \frac{\sqrt{dw}}{3^{1/4} 2\phi},
\]
which for our case yields $L_d = 85$ cm. This optimum is of course quite broad, so we are free to choose a more convenient value as long as it is with a factor of two or so; we have chosen $L_d = 60$ cm for the 16 MeV system.

Once the drift length is specified, there is another criterion which should be examined for the diagnostic to give unambiguous results, that the contribution to the measured emittance from the residual space-charge forces between beamlets is smaller than that due to the true uncorrelated angular distribution at the slits. Again assuming the slits are at a waist (this gives the highest estimate of the space-charge effect), we have

$$R_b = \frac{2I}{\gamma I_0 w_e} \frac{dL_d}{w_e n}.$$  

Again, this quantity must be much smaller than one. For our present design it is about 0.25, but it should be noted for Hartman’s measurements it was in fact greater than one.

The subject of slit scattering is a bit complicated, but a detailed calculation using EGS is not necessary if estimates that the signal to noise due to slit scatter is not of order 100 or less. Theoretical guidelines in this calculation have been developed by Courant (E.D. Courant, *Rev. Sci. Instr.* 22, 1003 (1951)) and Burge and Smith (E.J. Burge and D.A. Smith, *Rev. Sci. Instr.* 33, 1371 (1962)). Modifying Courant’s criterion for energy discrimination, we pick the effective depth of the maximum of the relevant slit scattered flux to occur when the multiple scattering angle is equal to the acceptance half-angle of the slits,

$$l_{\text{eff}} = L_r \left( \frac{21 \cdot d}{E(\text{MeV}) \cdot 2l} \right)^2$$

and the increase in effective slit width is given by

$$d_{\text{eff}} = \frac{2}{\sqrt{3\pi}} l_{\text{eff}}^{3/2} w_c,$$

where

$$w_c^2 = \frac{A}{Z^2 \pi N_A \rho} \left( \frac{E}{2e^2} \right)^2 \ln(181 \cdot Z^{1/3})^{-1}$$
and $N_A$ is Avogadro's number. The minimum signal to noise for the detected beam intensity at the phosphor is therefore

$$\frac{S}{N} \geq \frac{\sqrt{3} \pi d_{w_c}}{2 d_{eff} l_{eff}^{3/2}}$$

For our case, this is greater than $10^4$. It should be noted however, that a misalignment of the slits can generate anomalously large slit scattering effects, and thus care must be taken to avoid this situation.

The layout of the emittance slits in the beamline, and their hardware design are shown in an Appendix. More care has been taken in the machining specifications for the slits, to ensure that they are flat over the entire surface parallel to the beam propagation. This is accomplished by electron discharge machining (EDM), and is essential for our slit systems (and others being developed for Argonne and Fermilab which have even narrower slit requirements). The slits are now mounted on a rotatable actuator which is driven by a stepping motor, eliminating the tedious job of aligning the slits by hand.

Also included as Appendices are the calculations outlined above, performed for the UCLA, ANL and Fermilab beam measurement systems.
Appendix Pages

(Not all pages are included in this web based PostScript version)
The UCLA rf photoinjector phase space measurement system.
Calculations needed for the design of the slit based emittance measurement system (MKS units).

\[ \begin{align*}
I' &= 150 & \text{Beam current} & I_0' &= 1.7 \cdot 10^4 & \text{Alfven current} & E_0' &= 0.511 \\
E &= 16 & \text{Beam energy} & \gamma &= \frac{E}{E_0} \\
\varepsilon_n &= 4 \cdot 10^{-6} & \text{Normalized emittance} (\text{desired to measure}) & \sigma &= 1.5 \cdot 10^{-3} & \text{Beam initial size} \\
\varepsilon &= \frac{\varepsilon_n}{\gamma} & \text{Physical emittance} & \phi &= \frac{\varepsilon}{\sigma} & \text{Angular divergence of beam at focus} \\
\rho &= 9 \text{ g/cm}^3 & \text{Minimum ionizing stopping power for iron} \\
R_0 &= \frac{2.1 \cdot \sigma^2}{I_0' \cdot \gamma \cdot \varepsilon_n^2} & R_0 &= 79.257 & \text{This is the ratio of the space charge to the emittance terms in the rms envelope equation before the beam goes through the slits} \\
d &= 10^{-5} \cdot 2.5 & \text{The slit width} \\
R &= \frac{R_0}{\sqrt{3} \left( \frac{d}{\sigma} \right)^2} & \text{This is the ratio of the space charge to the emittance terms in the rms envelope equation for the beamlets after the beam goes through the slits.} \\
R &= 0.013 & \text{Should be much smaller than one, or...} \\
d_m &= \varepsilon_n \cdot \sqrt{\frac{3}{2} \cdot \pi \cdot \gamma \cdot \frac{I_0}{I}} & d_m &= 3.511 \cdot 10^{-4} & \text{Maximum tolerable slit width, should be many times actual slit width.} \\
I &= 5 \cdot 10^{-3} & \text{Depth of stopping iron.} & L_r &= 1.4 \cdot 10^{-2} & \text{Radiation length of Fe.} \\
\eta &= \frac{\text{ded}x}{E} & \eta &= 0.422 & \text{Fraction of energy lost in stopper, and} \\
\theta &= 21 \cdot \frac{E}{L_r \cdot \text{ded}x \cdot \left[ \frac{1}{1 - \eta} - 1 \right]} & \theta &= 1.032 & \text{rms multiple scatter these should not too much smaller than 1.} \\
a &= \frac{1}{d} \cdot \phi & a &= 0.017 & \text{Ratio of beam rms angle compared to slit acceptance, should be much less than 1.} \\
w &= 7.5 \cdot 10^{-4} & \text{Slit separation, should be smaller than initial beam size, much larger than slit width.}
\end{align*} \]
L_d := \sqrt{\frac{d \cdot w}{2 \cdot 3.025 \cdot \phi}} \quad L_d = 0.611 \quad \text{Optimum drift length to phosphor.}

\frac{L_d}{d} \sqrt{12} = 7.081 \quad \text{Actual drift distance}

L_d \cdot \frac{\phi}{d} \cdot 2 = 0.136 \quad \text{Overlap of beamlets at phosphor, should be much less than one.}

R_b := \left( \frac{2 \cdot 1}{10^{2 \cdot \gamma^2}} \right) \cdot \frac{L_d}{d} \cdot \frac{d}{w} \quad R_b = 0.09 \quad \text{Ratio of the maximum contribution to the rms angular width from the space charge, to the emittance part.}

\text{Slit scattering (Courant theory)}

Z := 26 \quad A := 26 \quad r_e := 2.8 \cdot 10^{-15} \text{ cm} \quad N_A := 6 \cdot 10^{23}

w_c := \sqrt{\frac{A \cdot 10^{-6}}{Z^2 \cdot \pi \cdot N_A \cdot r_e}} \cdot \frac{E}{2 \cdot r_e \cdot E_0} \cdot \frac{2 \cdot E_0}{1} \cdot \frac{1}{\ln \left( \frac{1}{181 \cdot Z^2} \right)}

w_c = 0.106 \quad l_{\text{eff}} := L \cdot \left( \frac{21 \cdot d}{E \cdot 2 \cdot 1} \right)^2 \quad \text{The effective length is the depth at which the rms angle of the slit scattered population is equal to the acceptance angle of the slits.}

\frac{d_{\text{eff}}}{d} = 1.436 \cdot 10^{-5}

\text{Signal to noise is}

\text{StoN} := \frac{d}{d_{\text{eff}}} \quad \text{StoN} = 6.964 \cdot 10^4
Hole is for 1/16 dowel, light press fit

Surfaces A, B, C will be refinished by Lordo Engineering using wire EDM to be flat and parallel to < 0.001

Pieces are to be assembled as shown in ASSEMBLY drawing.
Hole is for 1/16 dowel, light press fit

A Slit is to be 0.0020" deep. Corner radius not critical.

D Hole is for 1/16 dowel, light press fit

1 Surfaces A, B, C are to be flat and parallel to <<0.001

2 Pieces are to be assembled as shown in ASSEMBLY drawing

Material: 304SS
Quantity: 12 (ea)
Finish: 32 max

Emittance Measurement Slits:
STANDARD 50µm C Blocks

Design: Gil Travish

ALL DIM IN INCHES

SCALE: 1:5
SHEET2 OF3