Trapping of Background Plasma Electrons in a Beam-Driven Plasma Wake Field Using a Downward Density Transition

H. Suk, N. Barov, J. B. Rosenzweig and E. Esarey

Department of Physics and Astronomy, University of California, Los Angeles, CA 90095
*Center for Beam Physics, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

Abstract. Trapping of background plasma electrons by a beam-driven plasma wake field is studied as a new self-injection method. In this scheme, a short electron beam pulse is sent through an underdense plasma with a downward density transition and some background plasma electrons are trapped by the strong wake field due to the sudden increase of the wake wave wavelength at the density transition. Two-dimensional PIC (Particle-In-Cell) simulations show that a significant amount of plasma electrons are trapped and accelerated to a higher energy than the driving beam energy. Furthermore, the trapped-beam quality is fairly good. In this paper, the 2-D simulation results, dynamics of the trapped beam and the driving beam, and the proposed experiment for the UCLA Neptune Laboratory are described.

INTRODUCTION

In recent years, extensive studies have been performed to explore advanced accelerators based on a beam-driven plasma wake-field accelerator (PWFA) [1,2] or a laser wake field accelerator (LWFA) [3]. In both cases, injection is an important issue. In the LWFA, the plasma wavelength is in the 10 to 100 μm range for typical cases and a much shorter electron beam than the plasma wavelength should be injected to have a small energy spread. Injecting such a short external beam into the acceleration phase of the wake field by using lasers requires a femto-second timing accuracy which is beyond the current technology. Hence, to avoid such timing difficulties a few self-injection mechanisms were studied. They include the self-modulated laser wake-field acceleration (SMLWFA) [4], laser injection methods [5,6] and self-trapping of plasma electrons using wavebreaking of laser-induced wake wave in an inhomogeneous plasma density [7]. For the beam-driven PWFA, the plasma wavelength is rather larger (λp ~ a few mm for typical cases) so that the timing accuracy requirement is mitigated, but still a separate external beam should be generated to be injected.

To avoid such complexities, a new self-injection method for the PWFA was recently proposed by the authors of this paper [8]. In this scheme, a short (σz ~ c/ωp) electron beam pulse is sent through an underdense plasma (plasma
density $n_0 < \text{beam density } n_b$) with a sharp downward plasma density transition. When the beam passes through the sharp density transition, the wavelength of the plasma wake field increases suddenly so that some background plasma electrons are injected into the acceleration phase of the wake field. The injected particles are trapped and accelerated to a high energy by the wake field. In this scheme, the \textit{trapped electron beam} is focused in a nearly linear field of in a plasma-electron-free ion channel and the trapped beam does not have collisions with plasma electrons. Thus a good beam quality is expected and our simulation results verified that the new injection method can generate fairly good quality beams.

In this paper, we show the 2-D PIC (Particle-In-Cell) simulation results for trapping and acceleration of the background plasma electrons. They also include emittance and energy spread simulation results. In addition, transverse dynamics of the trapped beam, collision effects of the trapped beam with the background ions, and dynamics of the driving beam are investigated. Finally we propose an experiment for the UCLA Neptune Advanced Accelerator Laboratory to verify the trapping mechanism experimentally.

2-D PIC SIMULATIONS

In order to show trapping of the background plasma electrons, we performed 2-D simulations with the MAGIC code [9]. The plasma and driving beam parameters for the simulations are summarized in Table I. In the simulation, the driving beam has a bi-Gaussian density profile of $n_b(r, z) = n_{b,0} e^{-r^2/2\sigma_r^2} e^{-\xi^2/2\sigma_z^2} (\xi = z - \gamma_b t)$ and it propagates through an underdense plasma ($n_{b,0} = 2.4 n_0'$, $n_0'' = 3.4 n_0'$) with a downward density transition ($n_0'' = 0.7 n_0'$).

| Plasma densities | $n_0' = 5 \times 10^{13} \text{ cm}^{-3}$ for $z < 1 \text{ cm}$
| Plasma electron temperature ($kT_e$) | 3 eV
| Driving beam energy ($E_b$) | 50 MeV
| Beam density ($n_{b,0}$) | $10^4 \text{ cm}^{-3}$
| Beam size ($\sigma_r, \sigma_z$) | $k_p'' \sigma_z = 1$
| | $k_p'' \sigma_r = 0.56$ |
Figure 1 shows the phase space plot (r vs. z) of the background plasma electrons when the driving beam center is located at 1.76 cm and 13.76 cm, respectively. As shown in the plot, plasma electrons are almost completely expelled from the beam path due to the space-charge force of the driving beam and a plasma-electron-free ion cavity is formed behind the driving beam. In the plot, it is shown that some plasma electrons are injected into the acceleration phase of the wake field when the plasma wake wave passes the density transition (located at z=1 cm). The trapped particles are transversely focused in the plasma-electron-free ion cavity due to the plasma lens effect [10] and they execute betatron oscillations with a wavelength of 
\[ \lambda_\beta = \left( \frac{2\pi \gamma \beta}{n_0 r_e} \right)^{1/2}, \]
where \( r_e \) is the classical electron radius, \( \gamma \) and \( \beta \) are the relativistic factor and normalized velocity of the trapped particles. In the bottom plot of Fig. 1, the trapped beam is tightly focused after \( \lambda_\beta / 4 \) and separated from other plasma electrons. The trapped beam is so short (beam length \( <<1/k_p \) ) that the trapped beam does not develop the electron hose instability [11] and other instabilities during acceleration. However, if the trapped beam propagates farther in the ion channel, it begins to be affected by some slipped electrons from the driving beam, which is caused by depletion of the driving beam energy. This will be discussed later in detail.
Figure 2 shows the phase space plot for momentum of the trapped particles in Fig. 1. The trapped plasma electrons are observed to be rapidly accelerated to a relativistic energy and bunched in the first electron-free ion cavity. In the figure, note that a small amount of plasma electrons are also trapped in other rarefied cavities, but this trapping is due to wavebreaking which is completely different from the trapping mechanism in the first rarefied cavity. The figure shows that a lot more particles are trapped and accelerated in the first rarefied cavity than in other cavities. In this simulation, the total charge of the trapped particles in the first rarefied cavity is about 850 pC and the pulse duration is about 1 ps.

**FIGURE 1.** Phase space plot ($p_z$ vs. $z$) of the trapped plasma electrons in Fig. 1.

The trapped particles are almost linearly accelerated with a gradient of $dE/dz \equiv 430$ MeV/m in the beginning, but the energy is gradually saturated and then it begins to decrease. This is shown in Fig. 3. The saturation in the trapped beam energy is caused by depletion of the driving beam energy and this happens after the driving beam propagates a distance of $d = E_b/eE_0$, where $e$ is the electron charge, $E_b$ is the beam energy, and $E_0$ is the longitudinal electric field in the plasma wave. Figure 3 also shows the energy spread of the trapped beam during acceleration. The trapped particles have a certain energy distribution along the longitudinal direction,
with significant contributions from some stray particles in the head and tail parts of the beam. Hence, the particles in the head and tail parts should be cut off to have a smaller energy spread and emittance. Figure 3 shows the energy spread during acceleration for three different cases (20 %, 50 % and 90 % of the trapped particles). In the bottom plot of Fig. 2, for example, the rms energy spread $\Delta E / E_{\text{rms}}$ is 1.7 % for 20 % (Q=170 pC) of the trapped particles. In this case, the normalized rms emittance $\varepsilon_{n,\text{rms}}$ is estimated to be about 1 mm-mrad. Thus, the beam quality after cutting off the stray particles is fairly good.

![Figure 3](image)

**FIGURE 3.** Energy of the trapped plasma electrons for three different cases (90 %, 50 % and 20 % of the trapped particles).

**RADIUS OF THE TRAPPED BEAM**

When the trapped beam propagates in the wake field, they are accelerated longitudinally and focused transversely at the same time. During acceleration the trapped beam radius $R$ is governed by [12]

$$R'' + \frac{\gamma'}{\beta^2 \gamma} R' + \frac{\gamma''}{2 \beta^2 \gamma} R + K_0 (\gamma^2 f_e - 1) \frac{1}{R} + \frac{\varepsilon_{n,\text{rms}}^2}{\beta^3 \gamma^5} \frac{1}{R^3} = 0,$$  

(1)
where \( R^* \) and \( R' \) indicate \( d^2R/dz^2 \) and \( dR/dz \), respectively, \( f_e \) is the neutralization factor given by \( f_e = n_0 / n_{tb} \) (\( n_{tb} \) = trapped beam density), and \( \varepsilon_n \) is the normalized effective emittance. In the equation, \( K_0 \) is the generalized beam perveance defined by \( 2I/I_0(\beta\gamma)^3 \), where \( I \) is the current and \( I_0 \) is the characteristic current given by \( I_0 = ec / r_e \). As the trapped beam is accelerated, its radius changes. However, the radius gradually reaches an equilibrium (for a matched beam) as the energy of the trapped beam is saturated. In this case, the equilibrium radius \( \overline{R} \) is obtained from

\[
K_0(\gamma^2 f_e - 1) \frac{1}{R} = \frac{\varepsilon_n^2}{\beta^2 \gamma^2} \frac{1}{\overline{R}^3},
\]

which leads to

\[
\overline{R} = \left( \frac{r_e I}{e^2 c} + \sqrt{\left( \frac{r_e I}{e^2 c} \right)^2 + 2\pi \gamma^3 n_0 \varepsilon_n^2} \right) \frac{1}{2\pi \gamma^2 n_0}.
\]

If the trapped beam is emittance-dominated, i.e., if \( \varepsilon_n^2 / \beta^2 \gamma 2 \overline{R}^3 \gg K_0/\overline{R} \), Eq. (3) reduces to the well-known result \( \overline{R} = \left( \varepsilon_n^2 / 2\pi n_0 \gamma_e \right)^{1/4} \), while in the case of space-charge-dominated beams, it reduces to \( \overline{R} = (I / \pi n_0 e \beta \gamma^2)^{1/2} \). If the beam is not matched, the envelope of the trapped beam will oscillate around the equilibrium radius \( \overline{R} \).

**EMITTANCE AND ENERGY SPREAD CHANGE DUE TO COLLISIONS OF THE TRAPPED PARTICLES WITH THE BACKGROUND IONS**

When the trapped beam is accelerated in wake field, its emittance changes. This can be caused by several physical mechanisms, but multiple scattering of the trapped electrons with the background plasma ions and adiabatic damping during acceleration may be dominant. In the presence of uniform focusing the scattering with the background ions leads to an emittance growth that is linearly proportional to the longitudinal distance, while the adiabatic damping effect reduces the emittance. Hence, the overall emittance change is the sum of these two effects and it is given by [13].
\[
\begin{align*}
\left( \frac{d\varepsilon}{dz} \right)_{\text{tot}} &= \left( \frac{d\varepsilon}{dz} \right)_{\text{scatt}} + \left( \frac{d\varepsilon}{dz} \right)_{\text{acc}} \\
&= \frac{1}{\beta^4 \gamma^2} \left[ \frac{2\lambda \rho}{\alpha_f L_0} \left( \frac{m_e}{m_{\text{ion}}} \right)^2 - \beta \varepsilon_a \gamma' \right],
\end{align*}
\]  

where \( \rho \) is the ion mass density, \( \alpha_f \) is the fine structure constant defined by \( \alpha_f = m_{e} c r_{e} / \hbar \) (\( \hbar = \) Planck constant) and \( L_0 \) is the radiation length defined by \( d\gamma / dz = \rho (\gamma - 1) / L_0 \). In the case of high acceleration gradient, which is typical in the plasma wake field acceleration, calculations show that the adiabatic damping effect is several orders of magnitude larger than the emittance growth effect from the multiple scattering. Thus the emittance growth effect due to scatterings can be neglected.

Since the scattering effect does not lead to a significant emittance growth, it can be conjectured that the scattering effect will not increase the energy spread noticeably, either. To support this argument, we can calculate the ion cross section \( \sigma_i \). The nuclear radius \( r_n \) of an ion (with \( Z \) protons) for head-on collisions is given by \( r_n = 0.57 Z^{1/3} r_e \) [13], and based on this the mean free path \( 1 / n_0 \sigma_i \) can be calculated and shown to be several orders of magnitude larger than the typical plasma length (0.1 ~ 1 m) for the PWFA. Hence, the collision effect on the energy-spread growth can be completely neglected.

**DYNAMICS OF THE DRIVING BEAM**

Transverse dynamics of the driving beam for the previous simulation example is shown in Fig. 4. It shows that the head part of the beam expands due to the beam emittance, but particles in the middle and tail parts of the beam execute betatron oscillations in the rarefied ion cavity. In the beginning, they move coherently, i.e., they move as a whole. As the beam propagates further, however, the so-called phase mixing gradually occurs and the beam reaches a nearly equilibrium state. This state, which may be good for plasma particle acceleration, can not last long since the beam loses its energy continuously to excite the plasma wave and some beam particles eventually begin to slide to the acceleration phase of the wake field, as shown in Fig. 5. In this case, the driving beam electrons and trapped beam electrons are mixed together and they interact. This causes a rapid quality (represented by emittance and energy spread) deterioration of the trapped electron beam. Thus the trapped beam must be ejected before this happens. If the driving beam propagates further, more particles flow into the acceleration phase of the wake field, but soon these particles also lose their energy and have a slippage to the next period of the plasma wake field.
Eventually a kind of a longitudinal beam break-up occurs and this deforms the plasma wave form seriously.

**FIGURE 4.** Trajectory of the driving beam electrons in the plasma.
FIGURE 5. Momentum change of the driving beam electrons.

PROPOSED EXPERIMENT FOR THE NEPTUNE LABORATORY

In order to verify the trapping mechanism described above, we propose a proof-of-principle experiment for the UCLA Neptune Advanced Accelerator Laboratory. There are two plasma source options for this experiment. One is to use the existing argon discharge plasma source which was originally developed and tested for the UCLA underdense plasma lens experiment [14]. For the trapping experiment, minor modifications can be made to have a density transition in the plasma. This can be done by installing a mesh at the entrance of the beam-plasma interaction chamber. In this configuration, however, it may be difficult to have a sharp density transition due to plasma diffusion. In this case, another option can be considered. In this option, a rectangular (for example, 5mm x 10cm) high-power UV laser beam is illuminated on a lithium oven source transversely and a semi-transparent UV filter is placed to have an intensity step. Thus a lithium plasma column with a sharp density transition can be made. In this configuration, a high power UV laser is required and it is in the UCLA electrical engineering group already. A small (~10 cm long) lithium oven is also needed, but it can be built easily [15]. The Neptune beamline gives an electron beam
of about 16 MeV and this is high enough to do the proof-of-principle experiment. However, a high driving-beam charge requirement (>>1 nC) for the experiment will be challenging.

SUMMARY

A new self-injection method using a density transition for the PWFA was studied. The 2-D simulation results show that some plasma electrons are trapped and accelerated to a higher energy than the driving beam energy. Trapped beam quality for the simulation example was obtained and shown to be fairly good ($Q=170$ pC, $\Delta E_{rms}/E =1.7 \%$, $\epsilon_{n,\text{rms}}=1$ mm-mrad). Collisions of ions and the trapped beam was investigated and its effect on the beam emittance and energy spread growth was shown to be negligibly small. Finally two different options for a proof-of-principle experiment was proposed for the UCLA Neptune Laboratory.

ACKNOWLEDGMENTS

One of the authors (H.S.) would like to thank Dr. Chris Clayton at UCLA for helpful discussions on the rectangular UV beam method. Eric Esarey was supported by the U.S. Department of Energy, Contract No. DE-AC-03-76SF0098.

REFERENCES