Observations, Measurements and Applications of a Steady Collective Mode in an Electron Storage Ring

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A single bunch of charged particles in a storage ring is a system of $10^9$-$10^{11}$ coupled non-linear oscillators with radiative energy loss and compensated by the accelerating cavity. A stable collective mode in such system was induced and observed in the SPEAR storage ring using a turn-by-turn phase space monitor. New possibilities of using this mode as a tool to experimentally study non-linear dynamics in storage rings are discussed.

The motion of a single charged particle in a storage ring, including the effect of non-linear forces, can be studied by exciting an oscillation around the equilibrium orbit with a fast kicker and measuring the transverse position at two points in the ring over thousands or even millions of revolutions. This is equivalent to measuring a Poincare section at time intervals defined by the revolution period. The information can be used to determine important properties of the motion such as oscillation frequencies (betatron and synchrotron tunes), their change with oscillation amplitude and effects of non-linear resonances.

In practice, one can not measure the position of a single particle. Instead, we measure the center of mass of a bunch containing $10^9$ to $10^{11}$ particles having different oscillation amplitudes and frequencies. This fact complicates the situation. For instance, any frequency spread within the bunch can lead to decoherence of initial oscillation and a corresponding decrease in the center of mass amplitude. [1][2][3] This can limit our ability to accurately measure the single bunch oscillation amplitude and frequency.

Another problem can arise from self-induced forces, which can shift the frequency and introduce collective modes.

Under some conditions, discussed in [2][3], the bunch can lock into a single dipole mode and behave similar to a single particle with charge equal to that of the bunch. The growth rate of this mode can be set to nearly or exactly compensate for radiation damping. This allows us to explore a region of phase space for a much longer period of time.

Experimental results described in this paper were obtained in the electron storage ring SPEAR. The machine parameters relevant to this discussion are summarized in the Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>$E_s$</td>
</tr>
<tr>
<td>Revolution period</td>
<td>$T$</td>
</tr>
<tr>
<td>Horizontal tune</td>
<td>$\nu_x$</td>
</tr>
<tr>
<td>Vertical tune</td>
<td>$\nu_y$</td>
</tr>
<tr>
<td>Horizontal chromaticity</td>
<td>$\xi_x$</td>
</tr>
<tr>
<td>Vertical chromaticity</td>
<td>$\xi_y$</td>
</tr>
<tr>
<td>Number of electrons in a bunch</td>
<td>$N_b$</td>
</tr>
<tr>
<td>Transverse bunch size</td>
<td>$\sigma_x, \sigma_y$</td>
</tr>
<tr>
<td>Radiation Damping Time</td>
<td>$\tau_b$</td>
</tr>
<tr>
<td>Momentum Compaction Factor</td>
<td>$\eta_c$</td>
</tr>
</tbody>
</table>

Table 1. Parameters of SPEAR electron storage ring.

The transverse motion of each individual particle 'k' in the bunch can be described by the equation:

$$\dot{x}_k + \frac{1}{c\tau_x}x_k + k(s)x_k = f(x_k, y_k, s) + \left[ \text{Collective Forces} \right]$$

$$\dot{y}_k + \frac{1}{c\tau_y}y_k + k(s)y_k = g(x_k, y_k, s) + \left[ \text{Collective Forces} \right]$$

The damping decrement terms are included to model radiation damping. The functions $f$ and $g$ result from $x$-$y$ coupling and higher order terms in the Hamiltonian. To write the collective force terms explicitly, one needs some knowledge about the impedance of the accelerator. They contain forces from all particles at the location of the $k$-th particle including present (single turn) and previous (multi-turn) revolutions.

Under the condition that the oscillation amplitudes $x_k$ are much larger than the bunch size a similar equation can be written for the center of mass.

Single turn effects give rise to a head-tail instability [4][5] with growth (damping) rate given by
\[ \tau_{\text{Head-Tail}} = \frac{1}{l_b} \xi_{x,y} \cdot r_e \beta c N_b W_{\perp 0}(2l_b) \left( \frac{2\pi}{\gamma} \right)^2 \gamma \eta_{x,y}^2 \]  

(2)

Where \( W_{\perp 0}(2l_b) \) is the transverse wake function, \( r_e \) is the classical electron radius. Other terms in (2) are defined in Table 1.

The Beam Position Monitors (BPMs) are sets of button electrodes inserted in the vacuum pipe of an accelerator. The signals from these electrodes allow reconstruction of the transverse position of the bunch center of mass.

The architecture of the BPM data acquisition system used at SPEAR is shown on Figure 2. The raw button signals are linearly combined in the RF to produce signals proportional to the horizontal and vertical displacement, and a sum signal proportional to the total bunch charge. The signals are sampled at the peak value by ADC modules and stored for up to 128K turns.

The resolution of the single-turn BPM measurements (after time domain processing) is determined by the geometry of the buttons, electronics noise and ADC bit resolution. At SPEAR, we presently achieve 120 microns rms with 5mA in one bunch, which degrades as 1/current.

The response to a horizontal kick, when the chromaticity is set to its normal operating value (\( \xi_x = 0.8 \)), is shown in Figure 2. The fact that the bunch traces the same detuning curve for all kick amplitudes with error no greater than the natural tune variation of the machine [6] indicates that the amplitude of the center of mass is the same as the amplitude of individual particles (Figure 3).

Knowledge that the oscillation measurements accurately capture the dynamics of a ‘rigid body’ mode provides a powerful tool for accelerator diagnostics.

With a turn-by-turn system, tune shift with amplitude is usually measured by calculating the tune immediately after different amplitude kicks, before decoherence sets in.
Using the quasi-steady state dipole mode with coherent particle motion in SPEAR, however, it is possible to measure tune shift with amplitude following a single kick. Figure 3 shows that the resulting dependence is the same as if it was measured using many kicks. This technique avoids error introduced by slow variation of ring parameters, power supply ripple, or ground motion.

By adjusting the chromaticity and thus the growth rate of the dipole oscillation, one can observe particle loss in real time since the sum signal from the BPM processor is proportional to the instantaneous current.

It can be seen from Eq. 2 that when some fraction of particles is lost, the mode can become damped. As shown in Figures 7,8 after the oscillation amplitude has grown to some critical value (~18,000 turns), the enough particles are lost that the response pattern changes to exponential damping.

Tune variation due to the power supply ripple can be detected by measuring the tune at different phases with respect to the 60 Hz line voltage [6]. In this case the individual measurements were performed over a few seconds time interval. Non-ripple factors such as ground motion or temperature drift affected the accuracy. Using the quasi-steady state collective mode technique, ripple measurements will take only about 30,00 revolutions, or 25ms.
Figure 9. Fractional part of horizontal tune versus turn number x 100. Power supply ripple is evident.

When the beam is kicked, the instantaneous tunes will deviate from their linear value $\nu_x^0, \nu_y^0$ according to

$$\nu_x = \nu_x^0 + h_x J_x + \cdots$$
$$\nu_y = \nu_y^0 + h_y J_y + \cdots$$

The linear working point $\nu_x^0, \nu_y^0$ and kick strength can be chosen so that the bunch will start with the tune on one side of a resonance line and cross it as the amplitude changes. With the bunch motion locked in the dipole collective mode, it is possible to control the speed of resonance crossing by changing the damping (growth) rate via bunch current or chromaticity. Figure 10 shows the horizontal and vertical response to a horizontal kick near the resonance $2 \nu_x - \nu_y = 9$. The combined radiation and head-tail damping is set to $e$-fold in about 1,300 turns. The horizontal oscillations coherently couple to the vertical plane. Response becomes qualitatively different with slower crossing (damping time \~30,000 turns). Figure 11 exhibits horizontal response under these conditions. In this case no center of mass oscillations are observed in vertical plane (not shown). Slow crossing of a low order resonance effectively destroys the horizontal coherent mode.

Figure 10. Horizontal and vertical position (mm) versus turn number. Fast resonance crossing. Horizontal motion coherently couples to vertical motion.

Figure 11. Horizontal position (mm) versus turn number for slow resonance crossing. No coherent vertical oscillations are excited.

The coherent mode is also useful to study higher order resonances in the frequency map [7]-[9]. We reported on the first proof-of-principle measurement of frequency map in [10].

The frequency map ‘footprint’ on Figure 12 was obtained by numerical tracking with initial conditions chosen on a uniform grid in $J_x, J_y$ action space. For each initial condition the particle was numerically tracked for 1024 turns and the horizontal and vertical tunes were computed using NAFF. The linear working point of SPEAR ($\nu_x=7.166, \nu_y=5.26$) is located in the upper right corner. The non-linear resonance lines act as attractors or repellers for single particle motion in the tune space.

Figure 12. Frequency map of SPEAR (Simulation)

Four octupole magnets installed in SPEAR were used to introduce a positive horizontal tune shift with horizontal amplitude [11], so that a transverse kick would shift instantaneous tunes. To detect the presence of a resonant line we tuned the machine with the working point slightly above it. The evolution of the tunes was then followed after applying a horizontal kick.
Figure 13. Experimental evidence of the $3\nu_x + 2\nu_y = 32$ resonance crossing. Horizontal tune vs. turn number for 2
different nearby linear working points set approximately to
(a) $(7.158 ; 5.261)$ and (b) $(7.157 ; 5.261)$

In case (a), after the kick, the instantaneous tune
starts on the other side of the resonance line due
to the positive horizontal tune shift with
amplitude. The tune then crosses the resonance
line as the amplitude decreases. The flat part of
the upper graph is a manifestation of the
resonance crossing. In case (b), the linear tune is
chosen below the line so it is not crossed.


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