A PMQ-based, Ultra-short Focal Length, Final Focus System for Next Generation Beam-Radiation and Beam-Plasma Experiments

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Abstract. Next-generation advanced accelerators such as the PWFA, and beam-radiation interactions such as inverse-Compton scattering, depend on increased beam-density to achieve superior results. The photoinjector has enabled the production of high-brightness beams that are desirable for experiments with critical dependencies on bunch length and emittance. Along with the production of shorter and lower-emittance beams, comes the need to produce shorter focal-lengths (beta-functions). An approach to creating strong focusing-channels using high field, small-bore permanent-magnet quadrupoles (PMQs) has been followed by the authors. A focusing system using three PMQs, each composed of 16 Nd-Fe-B sectors in a Halbach geometry has been installed in the PLEIADES inverse-Compton experiment. As the magnets are of a fixed field-strength, the focusing system is tuned by adjusting the position of the three magnets along the beamline axis. This paper covers the details of the focusing system, experimental experience, and implications for future experiments with an emphasis on advanced accelerators.

INTRODUCTION

The availability of high brightness beams — made possible by the rf photoinjector — has opened new applications for electron sources. [1] These applications, e.g. driving a plasma wake-field accelerator (PWFA) [2] or providing an electron beam for an inverse-Compton scattering (ICS) X-ray source [3], are strongly dependent on production of very dense beams. It has been widely noted that the scaling of such applications to higher performance levels (higher field in the PWFA, brighter ICS source) requires shorter bunch lengths $\sigma_z$ and lower transverse emittances $\epsilon_{x,y}$, a situation that has strong implications for future development of electron beam sources themselves. These considerations have led to increased concentration on photoinjector optimization [4] and development of pulse compression systems. Much less emphasis has been placed, however, on the need to scale the beam focal systems down with the above relevant length scales. The focal system length scale is the minimum “$\beta$-function”, $\beta^*_z$, which is related to the transverse beam size at focus by the relation $\sigma_z^* = \sqrt[2]{\beta^*_z \epsilon_z}$. The difficulties in producing lower emittance beams further compels the use of small betafunctions.
The PWFA serves as an example of an application that is critically dependent on achieving small $\beta$-functions, and elucidates the relationship between $\beta^*_z$ and $\sigma_z$. The maximum longitudinal field (the “wave-breaking” field) achievable in a relativistic plasma wave scales with the plasma wavenumber $k_p$ as $E_{wb} = m_e c^2 k_p / e$. Further, in the PWFA it has been noted that if the beam is short ($k_p \sigma_z < 1$), then the field achieved in the plasma wave scales as $E \propto k_p^2 N_b$. [5] Both of these PWFA scaling laws indicate that to obtain large fields one should operate at larger $k_p$ and concomitant shorter bunch length $\sigma_z$.

The effect of using such large plasma wavenumbers is quantified by examining the matched $\beta$-function associated with the plasma’s ion-based focusing,

$$\beta_{eq} = \sqrt{\gamma / 2 \pi r_e n_0} = \sqrt{2 \gamma k_p^{-1}},$$

which is extremely short for low beam energy ($\gamma m_e c^2$) and large $n_0$. In present low energy experiments (FNAL, and previously at ANL [6]), $\beta_{eq}$ was already in the range of several mm. Next generation experiments plan to use short bunch lengths, higher plasma densities, and thus small spot sizes. [7]

In lower energy beams (<100 MeV), as are typical of state-of-the-art photoinjectors, it has not yet been possible to obtain such a small, matched $\beta^*_z \equiv \beta_{eq}$. In previous plasma experiments in this energy range, matching was either not achieved, or only achieved through tailoring the density transition at plasma entrance.

**APPROACHES TO SMALL BETA-FUNCTIONS**

One can consider three approaches to achieving small beta-function final focus systems: complex lattices of several magnets, short lattices of normal strength magnets with large magnification factors (i.e. entering with a big beam and exiting with a small one), and short lattices of strong magnets.

When one compresses the electron beam, either through a chicane or through velocity bunching [8], significant energy spread is introduced. In matching the beam into a plasma (or ICS), one must simultaneously avoid chromatic aberrations due to energy spread, and emittance growth due to residual space-charge effects in the beam if it is expanded for final focusing. Assuming one uses relatively weak normal-conducting electromagnetic quads, the final spot size (in the absence of an elaborate, long chromatically compensated final focus system) is limited by chromatic aberrations. These observations are illustrated by the following relation governing the ratio of final spot size $\sigma^*$ to initial $\sigma_0$, [9]

$$\frac{\sigma^*}{\sigma_0} = \sqrt{\frac{1 + \left(\frac{\beta_0}{\gamma}\right)^2 \left(\frac{2\sigma_{sp}}{p}\right)^2}{1 + \left(\frac{\beta_0}{\gamma}\right)^2 \left(1 + \left(\frac{2\sigma_{sp}}{p}\right)^2\right)}} \equiv \frac{2\sigma_{sp}}{p}.$$

$$\lim_{\frac{\beta_0}{\gamma} \ll \frac{\sigma_{sp}}{p}}$$
When the initial $\beta$-function $\beta_0 = \sigma_0^2 / \varepsilon$ is larger than the effective focal length $f$ of the final focus system, strong compression of the beam size is possible, up until chromatic aberrations begin to dominate, $\beta_0 / f \equiv p / 2\sigma_{\delta p}$, at which point the emittance grows rapidly and demagnification is limited to approximately twice the rms energy spread. These considerations strongly affect $\beta$-matching in previous and current PWFA experiments, and as the example below indicates, eliminate short normal-strength magnet lattices.

An example will serve to illustrate the technical implications of Eq. 2: A UCLA/LLNL collaboration is now operating an ICS source experiment termed PLEIADES [10], in which a 10-100 TW, sub-ps, 820 nm laser pulse is collided with a velocity compressed, sub-ps 50-75 MeV electron beam. With the present generation of relatively large aperture quads that UCLA employs, which have maximum gradient of 15 T/m over ~10 cm length, the minimum value of $f$ (effective focal length for a triplet) is roughly 50 cm, and thus optimum demagnification for $\sigma_{\delta p} / p = 0.5\%$ is only possible for $\beta_0 = 4$ m, or (with $\varepsilon_x = \varepsilon_n / \gamma = 0.1$ mm-mrad) $\sigma_0 = 2$ mm, and thus at minimum $\sigma^* \approx 30$ $\mu$m. This spot size estimate, which has been verified in simulation, is unacceptably large for the ICS source, which eventually demands a factor of better than 4 smaller spot size for nonlinear ICS interaction studies, and increased photon production. In addition, by allowing the beam to expand to $\sigma_0 = 2$ mm, space-charge forces will increase the emittance [4], giving an even larger final spot size: space-charge oscillations during the transport of an emittance compensated beam can cause emittance blow up if turbulent (non Brillouin) flow is created through large beam size variations.

This above considerations also preclude more elaborate approaches to elimination of the chromatic aberrations. At low energy, transport and final focusing systems must be compact and geometric aberrations must be avoided. Thus, short focal lengths are highly desirable. These and other considerations are valid for many high-energy applications as well, such as ICS production of polarized positrons, and linear collider (LC) final focusing.

It is clear that the use of much shorter focal length quadrupoles (or lenses of any type) is indicated in order to create $\beta$-functions small enough for modern high-brightness electron beam applications. One might consider superconducting quadrupoles or solenoids, but these devices are neither inexpensive, nor easy to build with the necessary small (cm-scale) dimensions. Exotic techniques such as plasma lenses introduce challenges (vacuum, scattering, beam parameter dependence) that are not desirable for many applications. On the other hand, permanent magnet-based quadrupoles (PMQs) can access high-field gradients, and have been under study intensive study recently in two arenas: LC main accelerator focusing — to mitigate the cost of the needed power supplies — and final focus magnets in both circular accelerators [11] and LCs [12], [13].
PERMANENT MAGNET QUADRUPOLES

By using an optimum, static intra-magnet configuration extremely large field gradients are accessible. Such a magnet configuration was originally introduced by Halbach [14]. This sectioned magnet configuration has been studied by the CLIC LC group, and has been implemented at CESR in the CLEO detector mini-β system [15].

A reasonable approximation of the PMQ field can be achieved by a Fourier analysis of \( M \) geometrically identical pieces with rotational invariance in the transverse geometry for the quadrupole solution, the fundamental strength is weakened from the pure quadrupole by the higher Fourier harmonics. The above model indicates, and 3D simulations confirm, that for \( M=16 \) segments, we obtain an almost pure quadrupole field with minimal deviation from the ideal field gradient (see Figure 1 which indicates the rotating field orientation resulting in a quadrupole field).

![Figure 1: Left: The field vector plot of the PMQ magnet, shown in 2D, but generated from the 3D code RADIA. Right: Photograph of the UCLA/PLEIADIES PMQ as built.](image)

With the above model in mind, we consider again the PLEIADIES ICS project where one must have a tightly focused electron beam collide with an oncoming tightly focused laser beam, and thus alignment tolerances are stringent, at the 5 micron level. PLEIADIES demands a range of operating energies, from 35 MeV to over 75 MeV. Because one needs the focal length of the system as a whole to be on the order of 10 cm, the individual PMQ length is chosen to be 1 cm. If one restricts, as is appropriate for a “triplet” configuration, the focusing phase advance for the 70 MeV case to be less than one-half \( \frac{k_q l_q}{l_q} \leq 0.5 \) in a single 1 cm lens, then a 600 T/m field gradient is deduced. This limit on focusing strength arises because alternating gradient focusing does not work well much beyond this level in the envisioned final focus configuration.

We are now able to, within the above limits, obtain a baseline PMQ design. The Halbach geometry is known to allow field gradients that may approach the ratio \( 2B_r / r_1 \), where \( B_r \) is the remanent field of the PM material, and \( r_1 \) is the pole inner radius. For the favored material NdFeB, \( B_r =1.22 \) T, and thus a 2.5 mm inner radius may produce over 600 T/m. In practice, 3D simulations (Radia) are used to calculate and optimize the final design (see Table 1 and Figure 1 right).
TABLE 1. Parameters of the PMQ magnets, as built.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic Material</td>
<td>NdFeB</td>
</tr>
<tr>
<td>Remnant Field</td>
<td>1.22 T</td>
</tr>
<tr>
<td>Length</td>
<td>10 mm</td>
</tr>
<tr>
<td>Bore Radius</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>Outer Radius</td>
<td>7.5 mm</td>
</tr>
<tr>
<td>Field Gradient</td>
<td>560 T/m</td>
</tr>
<tr>
<td>Effective Length</td>
<td>10.4 mm</td>
</tr>
</tbody>
</table>

Design and fabrication of the PMQ required error analysis and measurement. Sources of error include demagnetization, temperature effects, and geometric tolerances. Wedge angle, field orientation, bore radius and the attendant field magnitude errors also have to be considered. The error budget and tolerancing was accomplished in simulation, leading to specific fabrication requirements including, for instance, a method of eliminating roll errors by fabrication of a single long unit which was then sectioned. Confirmation of the field quality and gradient was accomplished through pulsed wire and hall probe measurements (see Figure 2).

![Figure 2](image)

**Figure 2**: Measurements and simulations are compared: (Left) A comparison of PMQ field profile measurements using a Hall probe to Radia simulation results; (Right) The effective magnetic length is plotted with measurements (squares) and simulations (line).

THE FINAL FOCUS SYSTEM

The high field PMQ allows for short focal lengths, and thus small beam sizes from high brightness beams. However, a final focus system that employs these magnets poses a design challenge. A lattice must be selected, and the strength of the lattice controlled. In addition, control of the input beam size is critical to avoid causes excessive beam sizes within the small aperture PMQ magnets.

Selection of the final focus lattice depends on the input beam energy, input beam size, and output sizes desired. We have used Elegant and TRACE3D to study the initial lattice design, sensitivity to errors, as well as for online tuning of the focusing lattice during beam runs. It can be shown that a more effective lattice of strong magnets for a final focus system is asymmetric, FDDFF, rather than the traditional
triplet (FDDF). This configuration is achieved, in practice, using 5 identical magnets held in 3 units (see Figure 3).

The need for extremely large field gradients makes the introduction of significant gradient tuning into a PMQ very challenging. Here, we have employed an alternative scheme that sidesteps these difficulties, by tuning the strength of the final focus array only through the longitudinal (z) positioning of the individual PMQs (again, see Figure 3). In this way, the issue of tunability is separated from both that of alignment, and that of achieving the highest gradient.

![Figure 3: A rendered CAD drawing of the final-focus assembly. The three lattice elements are shown on the three-rod alignment system, connected to the main vacuum flange (6” OD), with the linear actuators extending from the backside. The stepper motors and optical encoders are also shown in the back.](image)

**EXPERIMENTAL EXPERIENCE**

The PMQ system was installed in the PLEIADES [16] inverse Compton scattering (ICS) experiment in December, 2003 and high peak brightness, hard x-rays, were produced shortly thereafter.

A procedure of operation and tuning has been established and is revealing of how such final focus system is deployed: (1) Use quad scan to find emittance, Twiss parameters at input to matching electromagnet quads; (2) Use TRACE3D to determine matching quad settings for 200 µm input beam at PMQs; (3) Use TRACE3D again to determine PMQ spacing; and, (4) Tune focus by observing spot and adjusting spacing.

To date, the system has operated sufficiently well to deliver high x-ray fluxes at various beam energies, and electron beam sizes below 20µm. However, measurement of the input beam size is problematic (no proximate diagnostic), as is detailed measurement of the final beam spot (low resolution optics). Chromatic effects also
appear to be non negligible as the input beam emittance is a factor of 2 higher than the design value.

CONCLUSIONS

The strongest known adjustable PMQ system has been built and installed. Many mechanical and magnetic challenges have been addressed effectively, and the longitudinal positioning focusing system has proven successful. Finally, small bore PMQs appear well suited to lower energy experiments and final-focus schemes. These fixed strength PMQs may also appropriate for higher energy systems (i.e. NLC, LCLS, etc.).

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