Limits on production of narrow band photons from inverse Compton scattering

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Abstract. In using the inverse Compton scattering (ICS) interaction as a high brilliance, short wavelength radiation source, one collides two beams, one an intense laser, and the other a high charge, short pulse electron beam. In order to maximize the flux of photons from ICS, one must focus both beams strongly, which implies both use of short beams and the existence of large angles in the interaction. One aspect of brilliance is the narrowness of the wavelength band emitted by the source. This paper explores the limits of ICS-based source brilliance based on inherent wavelength broadening effects that arise due to focal angles, laser energy density, and finite laser pulse length effects. It is shown that for a nominal 1% desired bandwidth, that one obtains approximately one scattered photon per electron in a head-on collision geometry.

Keywords: Compton scattering, laser, X-ray, photon, bandwidth, emittance.

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INTRODUCTION

The use of inverse Compton scattering (ICS) in future light sources [1] spans many different fields of application and an extremely wide range of wavelengths. In the case of lowest scattered energy, in the angstrom spectral region (10’s of keV) one finds potentially revolutionary applications in medicine and materials characterization [2,3]. At the higher energy photons (few MeV), one finds application to examination of nuclear resonances for non-invasive nuclear materials detection; at the ~100 MeV range one may use ICS as an integral step in obtaining polarized positrons [4] for linear colliders; finally, ICS is proposed for converting electron energy at the hundreds of GeV scale to photon form in γ–γ colliders [5].

A common thread in the vast majority of applications is that one aims to maximize the number of scattered photons while simultaneously limiting the bandwidth of the scattered photons. For example, in medical imaging and therapy applications [6], a typical demand would be for $10^9 - 10^{10}$ X-ray photons in a 1% bandwidth. These two goals are at odds, however, as many of the techniques used in generating higher scattered flux cause either a range of red-shifts (e.g. strong focusing), or higher harmonics (nonlinear scattering). The purpose of this paper is to quantify the interplay between these competing design criteria, and identify the physical limitations on narrow-band ICS photon production, with the goal of informing both current ICS experiments [7,8] and planning for advanced ICS applications.

The basic geometry for ICS collisions that are nominally conducted head-on is shown in Fig. 1. The spread in incident angles resulting from the divergence of the
laser beam is indicated by $\theta_i$, and the final state scattering angle by $\theta_s$. The central frequency of the laser is indicated by $\omega_L$, and the nominal velocity of the electron is taken to be $v = \beta c$, where $\beta \approx 1$ and thus $\gamma = U_e/m_e c^2 = (1 - \beta^2)^{-1/2} >> 1$, i.e. the beam is ultra-relativistic.

Figure 1. Geometry for nominal head-on ($0^\circ$) ICS collisions, with spread in incident angle $\theta_i$ due to laser divergence shown, as well as final state scattering angle $\theta_s$.

At this stage, we assume for the sake of analysis that the electrons in the beam are all directed perfectly along the $z$-axis. This assumption allows us to concentrate on the angles due to the spread in laser photon direction, which is generally much larger than the spread in electron angles. As the spot sizes of the electron and laser beams are equal, this assertion is quantified by comparing the emittance of the electron beam with the effective “emittance” of the laser, and requiring $\varepsilon / \gamma \ll \lambda_L / 4 \pi$. For example, in the PLEIADES experiment at LLNL [9], even though $\varepsilon$ was quite large due to nonideal fields in the linac, this requirement is satisfied, as $\varepsilon = 1.6 \times 10^{-8}$ m, while $\lambda_L / 4 \pi = 6.4 \times 10^{-8}$ m. In fact, at PLEIADES, the electron beam was straightforwardly focused to a smaller spot than the laser [10] due in part to this disparity in “emittances”.

We will thus find that the estimate on ICS photons found within a certain bandwidth and angular spread is valid under this assumption. We must then add to the analysis a realization that the spread in electron beam angles leads to a breakdown in the direct correlation between lab scattering angle and wavelength. In order to obtain a certain bandwidth, a simple angular cut may not be sufficient. This point is discussed at the end of our analysis.

**ICS ANALYSIS IN THE THOMSON LIMIT**

Our ICS spectral analysis is performed in the Thomson limit, where the scattered photons as observed in the nominal beam rest frame do not have an appreciable wavelength shift (Compton shift) upon scattering. In this frame, the Lorentz transformation of the $(\omega, \vec{k} c)$ 4-vector from the lab frame yields the frequency $\omega'$, and the transverse and longitudinal components of the vector potential $k'_\perp$ and $k'_z$: 
\[ \omega' = \gamma(\omega_L - \beta k_L \cos(\theta_i)) = \gamma \omega_L (1 + \beta \cos(\theta_i)) \]

\[ k'_s = k_L \sin(\theta_i) \]

\[ k'_z = \gamma(k_L \cos(\theta_i) - \beta \omega_0 / c) = \gamma k_L (\cos(\theta_i) + \beta). \]

The Thomson limit is obtained \( \hbar \omega' \ll m_c^2 \), or \( \hbar \omega_L \gamma(1 + \beta \cos(\theta_i)) \ll m_c^2 \). For example, at the Neptune ICS experiment (which has \( \theta_i = \pi / 2 \)), the rest frame photon energy is \( (\lambda_e = 10.6 \text{ µm}, \gamma = 30) \) 3.6 eV. Thus we make the Thomson approximation that the frequency is independent of scattering angle, and \( \omega' = \omega = \gamma \omega_L (1 + \beta \cos(\theta_i)) \), and the wave-number components of the scattered photons are

\[ k'_s = \frac{\omega_L}{c} \sin(\theta') = k_L \gamma(1 + \beta \cos(\theta_i)) \sin(\theta') \]

\[ k'_z = k_L \gamma(1 + \beta \cos(\theta_i)) \cos(\theta') \]

where \( \theta' \) is the rest frame scattering angle. We note in the Thomson limit one may simply derive the emitted angular intensity (photon distribution) from the expression

\[ \frac{dP_s}{d\Omega'} = \frac{e^2 \nu^2}{4 \pi c^3} \cos^2(\theta'). \]

We will make use of this distribution in our calculation of the final scattered spectrum. In this regard, note also that the total Thomson scattering cross-section \( \sigma_T = 8 \pi \nu^2 / 3 \) is Lorentz invariant.

The frequency of the scattered radiation is found by a final Lorentz transformation to the laboratory frame,

\[ \omega_s = \gamma^2 \omega_L (1 + \beta \cos(\theta_i))(1 + \beta \cos(\theta')). \]

Equation 4 contains all needed information about the red-shifting of radiation which is emitted at finite angles. In order to interpret it, one needs to substitute for the rest frame angle \( \theta' \), however, in terms of the lab angle, which can be found by finding the wave vector components,

\[ k_{s,s} = k_L \gamma(1 + \beta \cos(\theta_i)) \sin(\theta') \quad \text{and} \]

\[ k_{s,z} = k_L \gamma^2 (1 + \beta \cos(\theta_i))(\cos(\theta') + \beta), \]

to give the expression

\[ \tan(\theta_s) = \frac{k_{s,s}}{k_{s,z}} = \frac{1}{\gamma} \left( \frac{\sin(\theta')}{\cos(\theta') + \beta} \right). \]

In the small-angle approximation Eq. 6 yields, simply, \( \theta_s = \theta' / 2 \gamma \).

The final, small-angle expression for the scattered frequency in the Thomson limit is thus

\[ \omega_s = 4 \gamma^2 \omega_L \left( 1 - \frac{\theta_s^2}{2} \right) \left( 1 - \frac{(\gamma \theta_s)^2}{2} \right). \]

The leading factor in Eq. 7 is the maximum Doppler shift given by the system kinematics, while the first and second correction terms give the effect of incident and scattering angles being nonvanishing. The final angle-induced red-shift is familiar from the FEL condition, where resonant emission occurs when the when emitted wave-front overtakes electron by \( \lambda_e \) in an undulator wavelength \( \lambda_u \), or \( \sim \lambda_L / 2 \) in the
Thomson system. In all systems where a relativistic electron is emitting radiation, the relative small-angle red shift is given approximately by \((\gamma \theta)^2/2\).

## PHOTON PRODUCTION AND BANDWIDTH

With these results in hand, we can parameterize the wavelength through the final angle. This approach has the benefit of also relating to the photon angular distribution, and thus the number of photons produced within a given bandwidth. In other words, the emission angle may serve as a proxy for the bandwidth. However, because of the finite spread in incident angles, and the fact that the red-shift due to this type of error observed on-axis is Doppler shifted as \((\gamma \theta_s)^2/2\), one may not simply limit the acceptance angle to clean the spectrum of emitted radiation. One must instead use other standard techniques, such as use of crystal-based monochromators [11] to pass the desired bandwidth. With this in mind, we can proceed to discuss the number of photons produced within a bandwidth using its functional dependence on the emission angle, leaving the extraction of only that bandwidth (if needed) for further discussion.

The efficiency of the photon production as a function of maximum emission angle (relative to the electron direction) is

\[
\eta_{\text{acc}}(\theta'_{\text{max}}) \equiv \frac{N_{\text{acc}}(\theta'_{\text{max}})}{N_{\text{total}}} = \frac{1}{2} \left(1 - \cos^3(\theta'_{\text{max}})\right) \approx \frac{3}{4} \theta'^2_{\text{max}} \quad (8)
\]

or in terms of lab angle, \(\eta_{\text{acc}}(\theta_s) \equiv 3(\gamma \theta_s)^2\). Written as a function of rms bandwidth, this acceptance efficiency is simply

\[
\eta_{\text{acc}}(\theta_{\text{max}}) \equiv 6\sqrt{3}(B W_{\text{rms}})_{\text{acc}}. \quad (9)
\]

There is also an expansion of the bandwidth of scattered light which is not apparent from the geometry shown in Fig. 1 — the spread in initial wavelength due to the finite character of the pulse train,

\[
(B W_{\text{rms}})_{\text{FT}} = \left(\sqrt{2}k_L \sigma_z\right)^{-1} = \frac{\lambda_L}{\sqrt{8\pi} \sigma_z}. \quad (10)
\]

This can be mitigated by choosing a longer laser pulse. However, there is a practical limit on the laser pulse length, in that it may not exceed the Rayleigh length,

\[
\sigma_z \leq Z_R = \frac{4\pi \sigma^2_z}{\lambda_L}, \quad (11)
\]
due to the free-space expansion of the laser over lengths exceeding this limit.

We must also examine the spread in incident angles due to the focusing of the laser which, as stated above, dominate the angles due to the electron beam emittance. Away from the laser focus, the rms spread in laser angle is

\[
\theta_{\text{l,rms}} = \frac{\sigma_z}{Z_R} = \frac{\lambda_L}{4\pi \sigma_z}, \text{ giving bandwidth } (B W_{\text{rms}})_{\text{foc}} = \left(\frac{\theta_{\text{l,rms}}^2}{2}\right) = \left(\frac{\lambda_L}{4\pi \sigma_z}\right)^2. \quad (12)
\]

A question arises as to whether this spread in angles actually displays itself inside of the laser focus, where the phase fronts are nearly flat, and do not show the angles seen outside of the focus. The answer is affirmative; one needs to consider the effect of the
Guoy phase shift, which is a function of \( z \), as \( \tan(\varphi(z)) = z/Z_R \). This dynamic phase shift gives a spread in longitudinal wave-number,

\[
BW_{\text{Guoy}} = \frac{\Delta k_z}{k_z} = \frac{1}{k_z} \frac{dq}{dz} = \frac{\lambda_u}{2 \pi Z_R} = \frac{\lambda_u^2}{8 \pi^2 \sigma_z^2},
\]

(13)
giving the same result as Eq. 12, when one considers the rms spread. This increase in laser bandwidth can be seen as arising from the free-space dispersion relation,

\[
k_z = \sqrt{(\omega_L/c)^2 - k_L^2},
\]

with \( k_z = \sqrt{k_L^2 - \sigma_z^2} \equiv k_L \left(1 - \frac{1}{2} \left(\frac{\lambda_L}{2 \pi \sigma_z}\right)^2\right) \).

We now adopt the restriction that, in order to maximize photon production, we choose the shortest Rayleigh range consistent with avoiding the “hour-glass” effect, \( \sigma_z = Z_R = 4 \pi \sigma_x^2 / \lambda_L \). Under this condition, we have

\[
\theta_{L,\text{rms}} = \frac{\lambda_L}{4 \pi \sigma_x} = \frac{\lambda_L}{4 \pi} \sqrt{\frac{4 \pi}{\lambda_L \sigma_z}} = \sqrt{\frac{\lambda_L}{4 \pi \sigma_z}},
\]

and so \( (BW_{\text{rms}})^{\text{FT}} = \theta_{L,\text{rms}}^2 = \sqrt{2} (BW_{\text{rms}})_F \),

(14)

and, remarkably, the bandwidth due to the Fourier transform of the laser pulse is nearly the same as that due to the spread in angles in the laser focus.

A final, critical, effect which induces spread in the scattered radiation spectrum is the nonlinearity of the motion due essentially to relativistic effects caused by the size of the laser field. A way to see this is that the electron has motion in the laser field with maximum induced angle of

\[
\theta = \frac{p_z}{p_0} \gamma \equiv a_L, \quad \text{where} \quad a_L = \frac{e E_L}{m_e c \sigma_z},
\]

(15)
giving a relative red-shift of \( (\gamma \theta)^2 = a_L^2 / 2 \). This shift is again familiar from FEL theory, if one substitutes the normalized undulator vector potential \( a_u \) for its counterpart corresponding to the laser, \( a_L \). Thus the spread in angles, due to the fact that the nonlinear motion-inducing electric field is not uniform, yields an rms contribution to the scattered bandwidth of

\[
(BW_{\text{rms}})^{\text{NL}} = \frac{a_L^2}{7.7}.
\]

(16)

This result is valid only for \( a_L << 1 \); for higher fields the motion takes on a “figure-8” character, and higher harmonics appear in the motion and the emitted spectrum [12].

**CONSTRAINTS ON LUMINOSITY AND PHOTON FLUX**

We are now in a position to calculate the photon luminosity \( N_\gamma = \mathcal{L} \sigma_T \), which gives the total scattered number of photons as \( N_\gamma = \mathcal{L} \sigma_T \). Maximizing the luminosity by our optimum choice of Rayleigh range, we have

\[
\mathcal{L} = \frac{N_L N_e^-}{4 \pi \sigma_x^2} = \frac{N_L N_e^-}{\lambda_L \sigma_z} = \frac{U_L N_e^-}{hc \sigma_z},
\]

(17)

where \( U_L \) is the total laser energy and \( h \) is Planck’s constant.

Use of \( U_L \) allows us to connect the luminosity to the laser energy density

\[
u_{\text{EM}} = \frac{N_L h \nu_L}{(2 \pi)^{3/2} \sigma z^2 / \alpha_x^2} = \frac{U_L}{\sqrt{\pi/2 \lambda_L^2 \sigma_z^2}},
\]

(18)
and thus the peak electric field value, as

\[
E_0 = \sqrt{\frac{2\eta_{EM}}{e_0}} = \sqrt{\frac{2U_L}{2\pi e_0 \lambda_L \sigma_z^2}}.
\]  

(19)

A constraint on the maximum tolerable \(a_L\) (and therefore the bandwidth due to nonlinear effects) thus also implies a limit on the total laser energy

\[
U_{L,\text{max}} = \frac{e_0 E_0^2 \Omega_\perp^2 \lambda_L}{(2\pi)^{1/2}} = \frac{k_l \sigma_z^2 \alpha a_{l,\text{max}}^2 m_c c^2}{(8\pi)^{1/2} \rho_e^2}.
\]  

(20)

We now arrive at an expression for the maximum achievable luminosity in terms of only the laser pulse length, wavelength, and normalized field,

\[
L_{\text{max}} = \frac{U_{L,\text{max}} N_e^-}{\hbar \sigma_z} = \frac{k_l \sigma_z^2 \alpha \sigma m_c c^2}{\sqrt{8\pi \rho_e^2 \hbar c}} N_e^- = \frac{a a_{l,\text{max}}^2 k_l \sigma_z}{\sqrt{8\pi \rho_e^2}} N_e^-,
\]  

(21)

with associated maximum number of photons per collision

\[
N_y = L_{\text{max}} \sigma_y = \frac{\sqrt{8\pi \alpha (k_l \sigma_z^2 a_{l,\text{max}}^2) N_e^-}}{6}.
\]  

(22)

This result, intriguingly, can be put in terms of the desired bandwidths, as

\[
N_y \approx 0.76 \frac{(BW_{\text{rms}})_{\text{acc}} (BW_{\text{rms}})_{\text{NL}} N_e^-}{(BW_{\text{rms}})_{\text{foc}}}.
\]  

(23)

Equation 23 is a powerful expression that can be used to guide design. The most obvious point that is deduced immediately from it is that one may use a smaller \((BW_{\text{rms}})_{\text{foc}}\) to obtain more scattered photons. This is because the only way to have more laser photons in the interaction given the assumed limit on energy density is to have a larger beam, and accompanying smaller \((BW_{\text{rms}})_{\text{foc}}\).

As an illustration of Eq. 23’s use, in many applications one desires a bandwidth on the order of 1%, so both \((BW_{\text{rms}})_{\text{acc}}\) and \((BW_{\text{rms}})_{\text{NL}}\) can be chosen maximally to be near 0.01. We shall see that a reasonable \(U_L\) is obtained when \((BW_{\text{rms}})_{\text{foc}} \sim 10^{-4}\). With these assumptions we have the remarkable result that \(N_y = N_e^-\). In order to evade this limit, one must be able to relax some assumptions concerning the laser, e.g. one may consider guiding it using a plasma channel [13].

**A SHORT DESIGN EXERCISE**

The emphasis in our analysis has been to constrain the laser, and so one must ask whether our constraints produce reasonable demands on the laser design. The first choice one must make in specifying the laser is in wavelength. From our analysis it is straightforward to see that the total laser energy scales in this choice as \(U_L \propto \lambda_L\), due to the limits on maximum tolerable field strength, and the simple fact that \(a_L \propto \lambda_L\). Thus, given our assumptions, one finds that contrary to the argument that large \(\lambda_L\) gives proportionally more photons per pulse for fixed \(U_L\), one is not permitted to use this laser with the same field value (intensity). We therefore assert that one should use a short wavelength laser, perhaps an 800 nm Ti:Sapphire or even a harmonic.
With the choice of $\lambda_L$, one then specifies $(BW_{rms})_{foc}$, which sets the dimensions of the beam unambiguously. Finally, one then chooses $(BW_{rms})_{NL}$, which then gives the energy density, and one arrives at the laser energy,

$$U_L = 0.1 \frac{(BW_{rms})_{NL}}{(BW_{rms})_{foc}} \frac{\lambda_L}{r_e} m_e c^2.$$  \hspace{1cm} (24)

Equation 24 shows explicitly that it is hard to exploit a larger beam size, as, and one is constrained by laser design and cost.

We have put together a full list of parameters to explore the utility of our approach in Table 1. We also add the parameters associated with an electron beam in order to specify the scattered wavelength (choosing a medical X-ray type example) and the emittance needed for the design. One can see that the beam spot size is rather small, with a Rayleigh range of under 0.5 mm. This type of focus may be obtained using permanent magnet quadrupoles, as done at PLEIADES [10]. Note that even in this, perhaps lowest electron beam energy $\epsilon << \lambda_L/4\pi$.

<table>
<thead>
<tr>
<th>Laser wavelength $\lambda_L$</th>
<th>800 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(BW_{rms})_{foc}$</td>
<td>$1.5 \times 10^{-4}$</td>
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<tr>
<td>Pulse length $\sigma_z(=Z_R)$</td>
<td>450 $\mu$m</td>
</tr>
<tr>
<td>$(BW_{rms})_{acc}$</td>
<td>0.01</td>
</tr>
<tr>
<td>Spot size $\sigma_x$</td>
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</tr>
<tr>
<td>$(BW_{rms})_{NL}$</td>
<td>0.01</td>
</tr>
<tr>
<td>Electrons $N_e$</td>
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<tr>
<td>Electron beam energy</td>
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<tr>
<td>Scattered photon energy</td>
<td>35 keV</td>
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<tr>
<td>Rms emittance $\epsilon = \epsilon_x / \gamma$</td>
<td>$2 \times 10^{-8}$ m-rad</td>
</tr>
<tr>
<td>Scattered photons in band</td>
<td>$3 \times 10^9$</td>
</tr>
</tbody>
</table>

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