Observations of low-aberration plasma lens focusing of relativistic electron beams at the underdense threshold

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Focusing of a 15 MeV electron bunch by a plasma lens operated at the threshold of the underdense regime has been demonstrated. The strong, 1.7 cm focal length, plasma lens focused both transverse directions simultaneously and reduced the minimum area of the beam spot by a factor of 23. It is shown through analytic analysis and simulation that the observed spherical aberration of this underdense lens, when expressed as the fractional departure of the focusing strength from its linear expectation, is \( \Delta K/K = 0.08 \pm 0.04 \). This is significantly lower than the minimum theoretical value for the spherical aberration of an overdense plasma lens. Parameter scans showing the dependence of focusing performance on beam charge, as well as time resolved measurements of the focused electron bunch, are reported. © 2010 American Institute of Physics. [doi:10.1063/1.3457924]

I. INTRODUCTION

The use of plasmas to focus electron beams has a long history. As early as 1922, Johnson1 described the electrostatic focusing of a continuous low-energy electron beam by beam-ionized gas within a cathode ray tube. A little over a decade later, Bennett2 showed that an electron stream could magnetically self-focus if it had sufficient current and its space charge had been neutralized by positive ions. While achievable electron beam parameters have changed radically since this early work, the mechanisms of passive (no external current) plasma focusing have remained largely the same.

Beginning in the mid-1980s, the high-brightness relativistic-beam physics community developed a great interest in electron and positron beam plasma lenses after Chen3 suggested that plasma lenses could form an immensely strong final focusing system for a linear collider. Such a system would take advantage of the ability of plasma lenses to produce radially symmetric focusing gradients equivalent to quadrupole lens gradients on the order of 1 MT/m, which exceeds the strength of conventional devices by many orders of magnitude.3 Additionally, it has been shown theoretically that adiabatic plasma lenses4 can overcome the synchrotron radiation-induced limit on final focus spot size.5 Consequently, there is great interest in using plasma lenses to achieve the small spot sizes and high luminosity necessary at the interaction point of future \( e^+e^- \) colliders such as the proposed international linear collider (ILC).

Plasma lenses for high-brightness relativistic beams are usually considered to operate in either the overdense \( (n_b \ll n_p) \) or underdense \( (n_b \geq n_p/2) \) regime, where \( n_b \) and \( n_p \) are the beam and plasma densities, respectively.6,7 A relativistic electron beam naturally tends to defocus as it propagates through a drift due to the beam’s finite emittance and space charge forces. The defocusing effect of the beam space charge is, however, largely balanced by the focusing magnetic self-forces generated by the beam current. The sum of the beam’s electrostatic and magnetic self-forces is equal to the electrostatic term multiplied by a factor of \( 1/\gamma^2 \), where \( \gamma \) is the Lorentz factor. Consequently, for highly relativistic beams \( (\gamma \gg 1) \) the cancellation of the self-forces is nearly perfect. In an overdense plasma lens the plasma electrons spatially configure so that the plasma ions cancel the beam space charge and thus allow the beam to focus under its magnetic self-forces. In general, these magnetic self-forces are not linear in distance from the beam axis \( (r) \), or uniform in distance along the beam axis \( (z) \), which leads to significant aberrations in the overdense case.6 In the underdense plasma lens the strong electric field created by the space charge of the electron beam ejects the plasma electrons from the beam region entirely, leaving a uniform ion column superimposed on a beam with balanced self-forces. It can be shown that the radial focusing force of this ion column is linear and given by \( F_{\text{radial}} = -2\pi n_i e^2 r \), where \( e \) is the electron charge. An underdense plasma lens is, therefore, theoretically free of spherical aberration in the limit that the ion column is as wide as the beam and the ions are immobile. The head of the beam is not focused in either plasma lens regime because of the finite response time of the plasma electron density distribution.

Focusing by overdense plasma lenses has been examined in several experiments. Nakashiki et al.8 were the first to show very modest overdense plasma lens focusing with a 20% spot size reduction. Hairapetian et al.9,10 subsequently demonstrated a factor of 5 beam spot demagnification with a well characterized overdense plasma lens and 3.8 MeV...
electrons. Ng et al.\textsuperscript{11,12} showed overdense plasma lenses focusing of 28.5 GeV electrons and positrons with lenses of up to 4 MT/m strength. None of these experiments sought to quantify the high aberrations of overdense regime.

While the overdense regime of plasma lenses has been relatively well explored, focusing in the low-aberration underdense regime has only been observed in other contexts. A series of experiments has examined the propagation of electron beams in underdense plasmas over long distances at relativistic\textsuperscript{13} and ultrarelativistic\textsuperscript{14–16} energies. In these experiments, the electron beam is focused by conventional elements to a size that will match the focusing properties of the ion channel formed in the underdense plasma and allows the beam to propagate at a fixed radius for many beam beta functions. The results of these past experiments establish that the aberrations of a strongly underdense focusing channel are low enough to be ignored when analyzing matched beam propagation for 3–12 beam beta functions, but they do not include significant demagnification of the beam by the plasma or a measurement of aberrations. Similar work has been done on the propagation of positron beams through long plasmas,\textsuperscript{17,18} which relies on a substantially different mechanism than in the electron beam case.

The lack of a previous underdense plasma lens experiment underscores the technical difficulty of this regime. For an underdense plasma lens experiment to be interesting it must clearly demonstrate low spherical aberrations, the key observable differentiating the underdense from the overdense regime, in a strong lens. From a technical standpoint, the need to clearly measure aberrations requires a strong underdense lens with large demagnification and precise knowledge of all the beam and plasma parameters. Previous overdense plasma lens experiments have shown the difficulty of simultaneously achieving these conditions. For low-energy beams it is difficult to get high enough beam density to allow for an underdense plasma lens of sufficient density, and therefore strength, to be interesting. While Hairapetian et al.\textsuperscript{10} achieved high-demagnification at 3.8 MeV, their accelerator could only produce $n_p \leq 5 \times 10^{10}$ cm\textsuperscript{-3}. Such a low density would dictate an underdense lens of less than 3 T/m strength, which is similar to the focusing power of a typical electromagnetic quadrupole magnet. It is much easier to obtain high density with high energy beams, but it becomes more difficult to produce a well-characterized short-dense plasma and achieve a high-demagnification underdense plasma lens. For example, Ng et al.\textsuperscript{11} produced a very strong lens and dense beam, $n_p \leq 2 \times 10^{16}$ cm\textsuperscript{-3}, but could only achieve a modest factor of 2 beam area reduction at the focus and had little direct control over the plasma density which was largely produced through direct ionization by the beam.

Choosing the operating point of a plasma lens within the underdense regime involves multiple factors. While the theory and beam propagation work discussed above both indicate that a strongly underdense lens will provide minimum aberration for currently achievable beam parameters, the situation changes when the expected parameters of ILC class beams are considered. Calculations predict that the assumption of immobile ions will be invalid in the case of ILC class beams passing through strongly underdense plasmas and that the ion motion will introduce large aberrations.\textsuperscript{19} While it has been suggested that manipulating ion mass may be a way to circumvent this problem,\textsuperscript{20} there are significant technical challenges with that approach. Since lowering $n_p$ mitigates the problem of ion motion, operating plasma lenses at the boundary of the underdense regime could be an attractive plasma lens scenario that offers minimum total aberrations for future linear collider beams.

The work presented here is the first ever experimental study of underdense threshold plasma lens focusing. Our observations show that the spherical aberrations of an underdense lens operated at the threshold of the regime, where $n_p = n_p/2$, are well below the lower bound on overdense plasma lens aberrations. This threshold for the underdense mode was initially noted in simulations by Su et al.\textsuperscript{6} and is somewhat surprising in that, from an examination of only electrostatic forces, one would suppose $n_p = n_p$ should be the threshold. While a theoretical discussion of the criteria defining the underdense regime is beyond the scope of this paper, our experimental investigation provides empirical data on the threshold of the regime.

**II. EXPERIMENTAL SETUP**

In the experiment a 14.8 MeV electron beam is focused by a plasma lens of approximately Gaussian profile in $z$ with full width at half maximum (FWHM) of 23.5 mm and peak density $n_{p,peak}=4.9 \times 10^{12}$ cm\textsuperscript{-3}. This plasma lens is uniform in $r$ over the length scale of the beam transverse size, has an average focusing strength $K=2366$ m\textsuperscript{-2} over an effective length of $l=25.0$ mm, and a focal length ($f=1/Kl$) of 1.7 cm. In magnetic quadrupole units, the lens has an average focusing gradient of 116 T/m, which is about 40 times stronger than the magnetic quadrupole fields used to transport the beam to our experiment. This focusing gradient also rivals state-of-art quadrupoles of both the permanent magnet\textsuperscript{21} and superconducting\textsuperscript{22} varieties without the complications of either small bores or cryogenic systems.

The plasma lens was created using a direct current electrical discharge transversely offset from the beam orbit, as illustrated in Fig. 1. A bulk plasma column with an approximate width of 5 cm FWHM is produced using our pulsed argon discharge plasma source\textsuperscript{23} operating with an argon pressure of 2.1 mTorr, 200 V discharge potential (across both the plasma and the 0.22 $\Omega$ ballast resistor), and 250 A discharge current for 2 ms pulses. The generated plasma is allowed to diffuse toward the beam path under the weak confinement of a 53 G solenoidal magnetic field aligned with the plasma flow. A conical stainless steel barrier suppresses diffusion into the interaction area and a thin slice of the static plasma column is selected using a translatable mask with a 1.25 cm wide by 4.5 cm high slit. The $z$ profile of the plasma lens formed by this mask was measured using an electrostatic Langmuir probe\textsuperscript{24,25} consisting of the 430 $\mu$m long tip of a 380 $\mu$m diameter tungsten wire. The probe was mounted to move along the beam axis, which is about 2 cm from the mask, so that the actual plasma densities encountered by the beam would be measured. The plasma density measurements, as derived from the ion saturation current,
showed the plasma lens to have a constant width of $23.5 \pm 1$ mm over the range of interest with a peak density that varied from $3.9 \times 10^{12}$ to $4.9 \times 10^{12}$ cm$^{-3}$ as the mask was translated, Fig. 2. The effect of this slight density modulation on the focusing observations will be discussed later. The analysis of probe I-V curves indicated a plasma electron temperature of about 4 eV. The plasma parameters were observed to be very stable and reproducible from shot-to-shot. The standard deviation of the measured plasma density for shots with identical source parameters and probe position was 8%.

The translatable mask arrangement of the plasma source allows the distance between the plasma lens column and the fixed diagnostic screen OTR3, see Fig. 1, to be varied with altering the optical transition radiation (OTR) collection optics. An in-vacuum, 160 mm focal length, 2 in. diameter lens provides quasiparallel collimation of the OTR produced by the beam optics elements and the layout of the plasma interaction region.

FIG. 1. (Color online) Schematic of the plasma lens experiment beam line starting with the vacuum isolation window. The positions of the three optical transition radiation screens (OTR1–OTR3) and the integrating current transducer are shown along with the beam optics elements and the layout of the plasma lens experiment.

The electron beam for the experiment is provided by the Fermilab NICADD Photoinjector Laboratory (FNPL) facility. The FNPL injector is a 15 MeV electron source consisting of a normal conducting L-band rf gun with a Cs$_2$Te photocathode and a nine-cell superconducting booster cavity. After acceleration, the electron beam is propagated at a tight focus through a Cs$_2$Te photocathode and a nine-cell superconducting booster cavity. After acceleration, the electron beam is propagated at a tight focus through a 10 nm thick aluminum vacuum isolation foil and refocused into the plasma lens experiment. This robust and highly reliable foil was required due to the extreme sensitivity of the nine-cell superconducting cavity to contamination and rises in its nominal 10$^{-10}$ Torr vacuum. Strong scattering in the aluminum foil significantly increases the beam emittance and produces a smooth Gaussian distribution, see Appendix. The beam is transported 2.2 m from the vacuum foil to the center of the plasma lens as illustrated in Fig. 1.

### III. OBSERVATIONS OF UNDERDENSE PLASMA LENS FOCUSING

In order to understand the spherical aberrations of our plasma lens we made a series of round beam focusing measurements with different spacings between the plasma lens and screen OTR3. These experiments used a 14.8 MeV bi-Gaussian electron beam with initial dimensions $\sigma_r = 692 \pm 54$ $\mu$m, $\sigma_z = 656 \pm 51$ $\mu$m, charge $Q = 18.8 \pm 2.0$ nC, and pulse length $\sigma_t = 22 \pm 3$ ps. Conse-
ently the initial peak beam density was \( n_{b,\text{peak}} = 2.5 \pm 0.5 \times 10^{12} \text{ cm}^{-3} \). Uncertainties are dominated by shot-to-shot variation of the beam. Peak density of the plasma lens is \( n_{p,\text{peak}} = 4.9 \times 10^{12} \text{ cm}^{-3} \) so \( n_{b,\text{peak}} \approx n_{p,\text{peak}} / 2 \) setting the experiment just on the boundary of the underdense regime, in terms of beam and plasma peak densities, at the entrance to the plasma lens. This is a lower bound however since the beam is substantially denser compared to the edge of the Gaussian plasma column which it first encounters and since the lens is thick, the envelope equation and simulations show that the beam focuses significantly as it traverses the plasma lens. We can, therefore, unequivocally state that the beam focusing is governed by underdense dynamics at the threshold of the regime. When optimally focused, a beam waist with \( x_{\text{FWHM}} = 259 \pm 22 \, \mu \text{m} \) and \( y_{\text{FWHM}} = 423 \pm 41 \, \mu \text{m} \) is achieved as shown in Fig. 3. The demagnification factor of 6.29 in \( x \) and 3.65 in \( y \) means that the transverse area of the focused beam is reduced by a factor of 23. Note that the overall distribution of the plasma focused beam observed on OTR3 is not a single Gaussian but a superposition of the strongly focused core and unfocused head and tail of beam, each of which remains roughly Gaussian. By using the FWHM to characterize the focused beam we automatically exclude the unfocused portions of the beam (halo) which are visible as broad bases of the focused peaks in Figs. 3(c) and 3(d).

IV. ANALYTIC ANALYSIS OF LENS ABERRATIONS

The transverse dynamics of a propagating electron beam can be described using the rms beam envelope equation.\(^2^8\) In general, the envelope equation has terms that account for the influence of the beam emittance, beam space charge, and the effect of transverse focusing elements. The ratio of the space charge term to the emittance term in the envelope equation is given by \( k_{p,b}^2 = \omega_{p,b} \beta_{c} / \epsilon_{c} \), where \( k_{p,b} \) is the relativistic plasma wave number associated with the electron beam \( \omega_{p,b} = \gamma_{e} / c \), \( \beta_{c} = \sigma_{c}^2 / \epsilon_{c} \) is the beam beta-function in \( x \), \( \sigma_{c} \) is the rms beam size in \( x \), and \( \epsilon_{c} \) is the geometric beam emittance in \( x \).\(^2^9\) After the vacuum isolation window \( k_{p,b}^2 \beta_{c}^2 / 4 \approx 0.06 \) indicating that the beam is emittance dominated and the influence of the beam space charge on beam envelope evolution can be ignored. Therefore, the space-charge-free form of the rms beam envelope equation\(^2^8\) can be used,

\[
\frac{d^2 \sigma_{x}}{dz^2} + K \sigma_{x} = \frac{\epsilon_{c}^2}{\sigma_{c}^2},
\]

where \( K \) is the focusing strength of the lens. The equivalent equation for the other transverse direction can be written by substituting \( y \) for \( x \). For an underdense plasma lens \( K = 2 \pi r_{e} n_{p} / \gamma \), where \( r_{e} \) is the classical electron radius and \( \gamma \) is the Lorentz factor.\(^3^0\) Detailed predictions of the plasma lens focusing of the beam core can be made by solving Eq. (1) using a function \( n_{p}(z) \) that describes the shape of our thick plasma lens. The envelope equation can also be used to describe the focusing of lenses with aberrations.

FIG. 3. (Color online) Images of the unfocused (a) and focused (b) electron beam OTR displayed with the same scaled-to-intensity color map and a constant pixel aspect ratio of 1:1. In order to provide sufficient contrast, the unfocused image (a) is the integration of five beam pulses. The projected intensity of the focused and unfocused beams (normalized to 1 pulse) in the \( x \) axis (c) and \( y \) axis (d) is also shown.
general way to do this is to define an effective emittance 
\[ \varepsilon_{\text{eff}} = \sqrt{\varepsilon_0^2 + \beta_0 \varepsilon_0 \delta \theta^2} \]
for use in Eq. (1) which includes both the original emittance of the beam \( \varepsilon_0 \) and the extra angular spread resulting from the lens aberrations \( \delta \theta \). The aberration-induced angular spread can be defined as 
\[ \delta \theta = (\sqrt{\beta_0 \varepsilon_0 / f})(\Delta K / K) \]
where \( \beta_0 = (\sigma_0^2 / \varepsilon_0) \) is the beam beta-function at the lens entrance, \( f \) is the lens focal length, and \( \Delta K \) is the difference between the linear expectation for the lens strength and its actual value at \( x = \sigma \). Su et al.\(^5\) showed that the minimum spherical aberration of an overdense plasma lens focusing a Gaussian beam is \( \Delta K / K = 0.21 \). As stated previously, an overdense plasma lens can theoretically be spherical aberration-free \( \Delta K / K = 0 \) in the ideal limit.

Fitting the predictions of the envelope equation to the focusing data obtained in the round beam case, Fig. 4 reveals a great deal about the plasma lens and its aberrations. In order to facilitate comparison to the simulation work discussed in Sec. V, the beam initial conditions are simplified by assuming \( \sigma_y = \sigma_y = \sigma_0 = \sqrt{\sigma_0^2 + \sigma_y^2} / 2 = 674 \) \( \mu \)m and \( d \sigma_y / dz = d \sigma_x / dz = 0 \) at the lens entrance. Calculations of the unfocused beam parameters at the plasma lens location using the initial conditions at the vacuum foil and the envelope equation describing the beam transport indicate that the assumption of zero convergence is a valid approximation. It must also be noted that the manner in which we generated our plasma lens leads to slight variation in its parameters. In this case, the plasma lens was measured to have peak density that varied from \( 4.5 \times 10^{12} \) to \( 4.9 \times 10^{12} \) \( \text{cm}^{-3} \) over the range of the data points, see Fig. 2. The width of the lens was constant at \( 23.5 \pm 1 \) mm over the range of interest. An increase of the plasma density due to beam ionization of the background neutral gas was not a concern due to the small ionization cross section for argon at 14.8 MeV \((1 / \nu_{\text{ionization}} \approx 13 \text{ ns} \gg \sigma_t)\), where \( \nu_{\text{ionization}} \) is the collisional ionization frequency.\(^3\) Additionally, damping of the plasma response to the beam due to neutral collisions can be ignored since \( \sigma_t \nu_{\text{en}} = 2 \times 10^{-4} \), where \( \nu_{\text{en}} \) is the plasma electron-neutral collision frequency.\(^3\) The variation in lens parameters leads to a slight variation in the lens focal length and therefore a broadening of the observed depth of focus, as shown in Fig. 4.

While the focal length is determined by the known plasma lens parameters, the size of the beam waist is determined by the parameter \( \varepsilon_{\text{eff}} \), which was obtained by a best fit to the data. For the curves shown in Fig. 4, \( \varepsilon_{\text{eff},x} = 105 \) mm mrad and \( \varepsilon_{\text{eff},y} = 170 \) mm mrad, where we have followed convention and used normalized emittance \( \varepsilon_n = \beta \gamma e \). A value of \( \varepsilon_n = 87 \pm 14 \) mm mrad was measured by a quadrupole scan downstream of the vacuum isolation foil, see Appendix. Using the above expression for \( \varepsilon_{\text{eff}} \) we can immediately calculate \( \Delta K_y / K_y = 0.08 \pm 0.04 \) which is well below the overdense lens minimum of 0.21 and thus strongly indicative of overdense operation.

We were not instrumented to make an independent measurement of \( \varepsilon_{x,n} \), which makes the analysis of aberrations in this axis more uncertain. The focused beam spot is clearly larger in \( y \) than \( x \), Fig. 3(b). If we assume similar lens performance in \( y \) as in \( x \) \((\Delta K_y / K_y = 0.08 \pm 0.04)\) it implies \( \varepsilon_{y,n} = 160 \) mm mrad. Such an emittance asymmetry is quite likely considering that \( \varepsilon_{x,n} \) and \( \varepsilon_{y,n} \) were dominated by scattering in the vacuum isolation foil and that the beam spot symmetry could not be guaranteed during this interaction. The magnitude of \( \varepsilon_{y,n} \) implied is also plausible since measured values of \( \varepsilon_{x,n} \) ranged from 87 to 165 mm mrad over several days of the experiment. We therefore conclude that the \( y \) axis data are consistent with the \( x \) axis results.

The observed aberration of our plasma lens has a chromatic component. The small energy spread of the electron beam was generated primarily by longitudinal plasma wake fields. Experimental limitations lead us to operate in an accelerator-like mode with \( k_p \sigma = 2.75 \) rather than in the preferred plasma lens limit \( k_p \sigma \gg 1 \), where \( k_p \sigma = c / \alpha_p \) is the plasma skin depth. Simulations of the beam/plasma interaction indicate an induced beam energy spread of 2.5% full width. From the total energy spread it is straightforward to calculate \( \Delta K / K \), chromatic \( \leq 0.025 \), indicating that the observed aberrations are predominately spherical.
V. SIMULATION RESULTS

In addition to the analytic analysis of aberrations presented above, a series of simulations was performed with the two-dimensional particle-in-cell finite-difference time domain code MAGIC. While MAGIC simulations describe the beam in \((r,z)\) coordinates, the definition of the particle distribution is chosen such that \(\sigma_x = \sigma_y = \sigma_p\). This is the same choice of description made by Su and Rosenzweig in their theoretical plasma lens work. Direct comparisons can therefore be made between results for \(\Delta K/K\) from MAGIC, the analytical analysis presented above, and earlier theory.

The parameters used for the MAGIC simulations are the same as in the analytical analysis, including the variation of the plasma density along the beam orbit. The simulation effort concentrated on the well characterized \(x\) axis and therefore used \(\epsilon_{x,n}\). In general, the results of the simulations match the experiment reasonably well, see Fig. 5, although there is some ambiguity in their interpretation. Just like the data, the simulations show that the electron beam consists of a tightly focused core superimposed on the broad base of the unfocused beam halo at the focus. The data were analyzed by summing the values of the beam image pixels in the direction opposite to the axis of interest and finding the FWHM of the resulting intensity profile. This projection approach, along with the background subtraction of an image taken with the beam shut off, was used to enhance the signal to noise ratio. If the analysis of the simulation particles is made to mimic this projection technique, the result is the blue band displayed in Fig. 5, which generally falls outside the error bars of the measurement. It is not clear, however, if the beam spot images perfectly record every particle in the beam halo the way the simulation data does. As Fig. 3 shows, when the cameras were adjusted to record the focused beam spot with good dynamic range it was necessary to sum several images of the unfocused beam to get enough contrast for a good analysis. If the beam halo is not fully represented in the beam images, its ability to increase the FWHM of the projection will decrease correspondingly. In such a case, it would be more accurate to compare the projection analyzed data to the FWHM of the simulation particle radial distribution \(\rho(r)\) at the focus, the red band in Fig. 5, which ignores any broadening in the projected FWHM from the halo. This method does indeed provide a better match to the data that fits within the error bars. While it is not possible to reconstruct the exact mapping between the beam distribution and the image data, the match between data and simulation results is good enough using either the projection or radial analysis to provide confidence that the simulation is capturing the essential behavior of the plasma lens.

It is also possible to directly extract the lens aberrations from the plasma lens simulation. This was done by copropagating a group of high momentum test particles with the beam. These particles were spread throughout the center 5 ps slice of the beam at a sufficiently low density that they did not affect the lens dynamics. Each particle had zero initial transverse momentum and high enough longitudinal momentum to prevent its trajectory from appreciably deflecting during passage through the lens. Examining the average inward momentum gained by these test particles allows us to directly measure the aberrations experienced by the center section of the electron beam, Fig. 6. The 7% aberration, \(\Delta K/K=0.07\), found using this technique compares well to the value found using the analytic analysis. The fact that the simulation result yields a somewhat lower aberration value is consistent since the analytic analysis integrates over the whole focused beam. These aberration values, which lie in between the underdense ideal of \(\Delta K/K=0\) and the overdense minimum of \(\Delta K/K=0.21\), support the idea that this plasma lens operates at the threshold of the underdense regime.
VI. VARIATION OF PLASMA LENS PERFORMANCE WITH BEAM DENSITY

As discussed above, the strength of an underdense plasma lens, \( K = 2\pi \gamma n_p / \gamma \), is independent of the beam density while the strength of an overdense plasma lens is not. Since the plasma lens examined in this experiment operates at the threshold of the underdense regime it is logical to expect that changes in the beam density will affect the lens performance. The analysis of the focusing data clearly shows such a relationship.

Beam density \( n_b \) is not a directly observable quantity. Furthermore, due to the fluctuations of beam parameters in this experiment and the fact that all the density-relevant parameters cannot be measured simultaneously, only the average density over many shots can be calculated. For this reason we use beam charge \( Q \), which was precisely measured on every shot, in lieu of density for our parameter scan.

Examining the variation of the unfocused electron beam spot size and duration with charge shows that shot-to-shot variation overrhwems any correlations with beam charge over the range examined. Therefore, while the exact density of a particular shot cannot be calculated, it is reasonable to assume that the beam density scales roughly with its charge on average.

Only shots with high charge, \( Q \geq 16.3 \) nC, were used in the focusing analysis presented above. By looking at the correlation between beam charge and minimum focused beam spot size in \( x \) for all the collected data, we can immediately see that the lens appears to lose strength and/or has larger aberrations at lower charges, see Fig. 7. Plotting the correlation between charge and the focused beam aspect ratio is also useful, see Fig. 8. The aspect ratio varies as the charge changes due to the competition between the beam emittance and the lens aberrations. One would expect that an aberration free lens will focus a round beam to a waist with an aspect ratio equal to the emittance ratio \( \sigma_y^2 / \sigma_x^2 = \epsilon_y / \epsilon_x \), where a superscript asterisk is used to denote the beam spot at the focal point of the lens. In this experiment, \( y_{\text{FWHM}} / x_{\text{FWHM}} \approx \sigma_y^* / \sigma_x^* = \epsilon_y / \epsilon_x = 160 \) mm mrad \( / 87 \) mm mrad = 1.8, which is very close to the aspect ratio we actually observed at high charge. If aberrations are introduced into the system, however, they will have the effect of rounding out the focused beam spot. The rounding effect can be seen by writing out the approximate form for the aspect ratio of the focused beam spot when aberrations are present,

\[
\frac{\sigma_y^*}{\sigma_x^*} \approx \sqrt{\left(\frac{\epsilon_y}{\sigma_y}\right)^2 + \left(\frac{\Delta K}{K}\right)^2},
\]

As the aberration term becomes dominated, the focused beam spot approaches the limit \( \sigma_y^* / \sigma_x^* = \epsilon_y / \epsilon_x = 1 \) in this experiment, the focused beam aspect ratio should change from about 1 to 1.8 as the plasma lens transitions from an aberration dominated regime to an emittance dominated regime. While the data appear to follow this trend, a quantitative analysis would require a derivation of the relationship between \( Q \) and \( \Delta K / K \) in the transition region between underdense and overdense regimes, as well as three-dimensional simulations of the beam focusing, which are beyond the scope of this paper.

VII. TIME RESOLVED MEASUREMENTS

A series of time resolved measurements of the electron beam were made using a 2 ps resolution streak camera. While the streak camera was typically used to measure \( \sigma_y \), we also added a second lens to the optical transport in an effort to image the OTR onto the streak camera slit and observe the variation of the beam radius in time with plasma focusing. An example of the images recorded on the streak camera, along with an analysis of many shots, is shown in Fig. 9. The data points represent 6.7 ps wide slices of the beam. As expected, the intensity profile of the focused beam...
in the time domain remains roughly Gaussian both with and without plasma focusing. In the transverse dimension, which is a mixture of $x$ and $y$ since the transport was not designed to maintain orientation, the beam is radially larger at the head than in the middle or tail when focused and fairly uniform without focusing. This time domain behavior is in general agreement with theoretical predictions of plasma focusing. The focused diameter of the beam core observed on the streak images is also in reasonable agreement with the value measured using the OTR screen CCD camera. There is, however, clearly clipping of the signal in the transverse direction as evidenced by the FWHM measurement of the unfocused beam on the streak camera being a third of the measured value on the CCD camera. This problem was discovered during the postexperiment analysis. To our knowledge, there were no limiting apertures to explain this effect. The clipping may be the result of losses over the long 7 m light transport to the streak camera via two lenses and six mirrors, which might have reduced the low intensity edges of the beam below the detection threshold. Without detailed knowledge of the problem with the measurement, it cannot be corrected after the fact. Therefore, while the streak camera data on the beam focusing are qualitatively consistent with the rest of the focusing data, no quantitative conclusions can be taken from it.

VIII. CONCLUSIONS

We have measured extreme demagnification of a high-brightness electron beam by a strong plasma lens operating at the threshold of the underdense regime for the first time. Multiple analyses of the data support the conclusion that the lens is operating at the boundary of the underdense regime, as anticipated from the beam and plasma densities ($n_b=n_p/2$). It is also shown that this lens has lower aberrations than the theoretical minimum for overdense lenses even though it operates at the underdense boundary. We conjecture that the lens has a reasonably well formed ion column in this threshold regime due to the additive effect of the electrostatic forces and the mutual repulsion between the beam and plasma return currents. Operation at the boundary of the underdense regime may be an attractive plasma lens scenario since it combines low-aberration with minimal beam density; lowering $n_b$ may mitigate the problem of ion motion\textsuperscript{19,20} for ILC class beams. Further investigation, both theoretical and experimental, should be pursued to understand the dynamics of plasma focusing in the underdense boundary regime.

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APPENDIX: CHARACTERIZATION OF THE ELECTRON BEAM EMITTANCE

Since the beam spot size observed at the focus of the plasma lens is determined by both the lens aberrations and beam emittance, an independent measurement of the beam emittance is vital for deconvolving the lens aberrations. We used the well established technique of quadrupole scanning\textsuperscript{36} to determine the emittance of the electron beam entering the plasma lens. Quadrupole scans were preformed using the second quadrupole after the vacuum isolation foil and OTR2, see Fig. 1. The OTR2 screen was tilted at 45° to the beam axis so that front surface OTR could be observed. Measurements of the horizontal emittance $\epsilon_x$, which requires focusing the beam into a vertical stripe, could be made with good accuracy in this geometry. Two examples of quadrupole scans taken during the experiment are shown in Fig. 10.
Accurate measurements of $\epsilon_y$ were, however, not practical using the tilted OTR2 screen due to the horizontal beam stripe’s short depth of focus. While the quadrupole scan is a well-established emittance measurement technique, it is not suitable for all beams. Anderson et al.\cite{Anderson} showed that quadrupole scans can give erroneous results when used to measure high-brightness electron beams for which space-charge forces are comparable to emittance effects. The parameter space in which quadrupole scans are an accurate way to measure beam emittance can be defined using the relativistic plasma wave number associated with the electron beam $k_{p,b} = \alpha_{p,b}/c \equiv \sqrt{4\pi n_i/\gamma}$. Accurate quadrupole scans can be expected under the conditions $k_{p,b}L_d < 1$ and $k_{p,b}\beta_x < 1$, where $L_d$ is the drift distance between the center of the quadrupole and the OTR screen, and both $k_{p,b}$ and the beam beta-function $\beta_x$ are computed at the quadrupole. For both the quadrupole scans shown in Fig. 10 and $L_d = 0.438$ m. Since the other parameters of the beams measured in Figs. 10(a) and 10(b) are comparable, the detrimental influence of space charge is greatest in the lower emittance case of Fig. 10(b). For the Fig. 10(b) measurement $\gamma = 29$, $(n_i) = 1.1 \times 10^{11}$ cm$^{-3}$, and $\sigma_z = 2$ mm so that $k_{p,b}L_d = 0.18$ and $k_{p,b}\beta_x = 0.54$. Consequently, our quadrupole scans fall well within the valid parameter space. Additionally, our quadrupole scans have a high degree of symmetry about the minimum, as shown in Fig. 10(a), which is another indicator that they give accurate emittance measurements.\cite{Anderson}

The measured emittances also agree well with expectations derived from calculations of scattering in the vacuum isolation foil. As discussed in Sec. II, there is a 10 $\mu$m thick aluminum vacuum isolation foil separating the 2.1 mTorr plasma lens experiment from the rest of the beam line. The amount of emittance growth produced by the beam’s interaction with the foil can be calculated statistically. The distribution of deflection angles produced by multiple Coulomb scattering is approximately Gaussian for small deflection angles. Assuming this small angle limit and that a zero divergence electron beam with $x' = 0$ enters the foil, the rms angle of the beam electrons emerging from the foil, $\sigma_{x'\text{foil}}$, is given by

$$\sigma_{x'\text{foil}} = \frac{13.6 \text{ MeV}}{\beta c p} \frac{x}{X_0} \left[ 1 + 0.038 \ln \left( \frac{x}{X_0} \right) \right],$$

where $p$ is the particle momentum, $\beta c$ is the particle velocity, $z$ is the charge number for the particle ($z = 1$ for the electron), $x$ is the thickness of the foil, and $X_0$ is the radiation length of the material.\cite{Kellerer} The radiation length of aluminum is 8.57 cm.\cite{Kellerer} For the case shown in Fig. 10(b) the electron beam energy is 14.8 MeV. Using Eq. (A1) to calculate the angular spread that a beam of this energy gains by passing through the 10 $\mu$m aluminum window gives $\sigma_{x'\text{foil}} = 6.5$ mrad. In order to translate this angle into an emittance, recall that the normalized beam emittance in $x$ is given by $\epsilon_{x,n} = \beta \gamma \sigma_x \sigma_{x'}$. Once again for the case shown in Fig. 10(b), the approximate beam spot expected at the foil, as extrapolated from measurements at OTR1, is $\sigma_{x,\text{foil}} = 450$ $\mu$m. Combining these facts gives an estimate for the emittance resulting solely from scattering in the foil, $\epsilon_{x,\text{foil}} = \beta \gamma \sigma_x \sigma_{x'} \sigma_{x'\text{foil}} \approx 85$ mm mrad, which is nearly equal to the measured value $\epsilon_{x,n} = 87 \pm 14$ mm mrad. By contrast, the 2.1 mTorr argon behind the vacuum window has a radiation length of $4.6 \times 10^9$ cm and has a negligible effect on beam emittance over the $\sim 2$ m propagation length $\epsilon_{x,n,Ar} = 0.9$ mm mrad.\cite{Kellerer} Clearly, scattering in the foil is the dominant factor in the beam emittance. This conclusion is consistent with the observation of much lower emittances before the foil. Spoiling the beam emittance by passing it through a thick foil, while generally undesirable, does have the positive effect of thermalizing the beam and producing a smooth Gaussian distribution.

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