Improving the coherence of SASE FEL light pulses has been a goal of research for many years. SASE FELs operate on the principle that a coherent electromagnetic wave can be emitted and amplified many orders of magnitude by the collective beam-radiation instability that propagates through a periodic static magnetic undulator. The net magnetic wave is amplified and emitted many orders of magnitude by the collective beam-radiation instability that propagates through the periodic static magnetic undulator. This instability is known as the Waltz instability, and it is the result of the interaction between the electron beam and the magnetic field of the undulator. The Waltz instability is driven by the phase mismatch between the electron beam and the magnetic field of the undulator. When the phase mismatch is large, the amplification process is exponential, and the radiation intensity increases exponentially with the number of undulator periods. This exponential growth of the radiation intensity is known as the gain of the FEL. The gain of the FEL is determined by the collective beam-radiation instability and the longitudinal coherence of the electron beam. The longitudinal coherence of the electron beam is determined by the electron beam phase space and the electron beam energy chirp. The electron beam phase space is determined by the electron beam electron energy spread and the electron beam energy chirp. The electron beam energy chirp is determined by the longitudinal acceleration of the electron beam and the longitudinal energy spread of the electron beam. The longitudinal acceleration of the electron beam is determined by the longitudinal electric field of the undulator and the longitudinal energy spread of the electron beam is determined by the longitudinal distribution of the electron beam. The longitudinal energy spread of the electron beam is determined by the longitudinal distribution of the electron beam and the longitudinal energy spread of the electron beam is determined by the longitudinal acceleration of the electron beam and the longitudinal energy spread of the electron beam.
the physics of this scheme at longer wavelengths. In this regard, the frequency-resolved optical gating (FROG) diagnostic and pulse reconstruction technique has proven to be effective in analyzing ultra-fast light sources in the optical and near-infrared wavelength regions. In this letter, we report a time-domain measurements of near-single-pulse radiation produced from a single pass SASE FEL amplifier operating under the e-beam energy chirp and undulator tapering conditions discussed above, by utilizing an advanced transient-grating (TG) FROG diagnostic. We compare these results with fully three-dimensional time dependent numerical simulations obtained from the code GENESIS.13

The experiment, with e-beam energy chirp and compensating undulator taper, was conducted at the SPARC (Sorgente Pulsata Auto-amplificata di Radiazione Coerente) FEL Facility in Frascati.14 High brightness electron beams are created at SPARC using an injector consisting of a 1.6 cell rf photocathode gun, followed by three traveling wave linac sections, two of which are embedded in focusing solenoids. Longitudinal e-beam compression is obtained by running the first linac section off crest near the zero-crossing phase, where the first solenoid is used to optimize emittance compensation.15 This method, termed velocity bunching,16 gives an increase in the peak current while leaving a residual energy chirp in the electron beam longitudinal phase space.17,18 Two quadrupole triplets along with a dipole based magnetic spectrometer and RF deflecting cavity allow measurement of time-projected and time-resolved (slice) e-beam parameters, which are summarized in Table I. Transport optics match the beam into the undulator, which is comprised 6 independent, variable gap sections. The undulator sections were discretely tapered through gap adjustment in accord with the above prescription for \( K(z) \), and for a radiation spike propagating with group velocity near the speed of light. After undergoing exponential gain and entering saturation (see Table I for relevant parameters), the FEL light exiting the last undulator section was directed to a diagnostic station where it was input to the FROG device.

The TG FROG diagnostic used to extract the longitudinal profile was constructed with the unique capabilities and challenges of measuring FEL output in mind,19 particularly in its extendibility to shorter wavelength operation. It was based on a geometry initially proposed and demonstrated by Lee et al.20 The diagnostic geometry as constructed is shown in Fig. 1. The expression for the FROG trace for this particular geometry is \( \hat{F}_{\text{FROG}}(\omega, \tau) \propto \int E(t) E^* (t-\tau) \exp(-i\omega t) dt \). Iterative Fourier transform algorithms are used to retrieve the complex electric field, \( E(t) \). While the TG FROG concept has previously been studied,21,22 it was not used in the FEL context. Here, we emphasize that the geometry implemented has minimal alignment degrees of freedom and operates on a single-shot basis; these key features help simplify complex measurements at a FEL.

At this point, it is useful to discuss relevant previous experimental results in chirped SASE FELs and the applications of FROG diagnostics to provide context for our results. Andonian et al.23 studied the effects of a chirped e-beam on the spectral characteristics of the output radiation and found the resulting light exhibited an anomalously large bandwidth. Li et al.24 used a negatively chirped e-beam, where the electrons in the head of the bunch had a higher energy than that of the electrons in the tail, to compensate for the light’s intrinsic positive chirp that naturally arises from the SASE process. Velocity bunching, however, results in a positively chirped electron beam, which serves to reinforce this intrinsic chirp. In the present experiment, the undulator tapering serves to compensate for both the intrinsic SASE chirp as well as the e-beam chirp, while simultaneously compensating for the electron energy loss to the FEL radiation. This

![Diagram](image)

**FIG. 1.** Schematic of transient-grating FROG diagnostic: (a) beam expander; (b) input mask, cylindrical lens, Fresnel prism; (c) nonlinear optical medium, output mask, knife edge slit; (d) focusing optics, diffraction grating; (e) CCD camera.

**TABLE I.** Experimental e-beam, undulator, and radiation parameters.

<table>
<thead>
<tr>
<th>Parameter Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge ( Q )</td>
<td>250 pC</td>
</tr>
<tr>
<td>Peak current ( I )</td>
<td>264 A</td>
</tr>
<tr>
<td>Mean e-beam energy ( E )</td>
<td>113.1 MeV</td>
</tr>
<tr>
<td>e-beam energy chirp ( \sigma )</td>
<td>(-2.5\text{keV}/\mu\text{m})</td>
</tr>
<tr>
<td>rms slice energy spread ( \sigma_\gamma )</td>
<td>(5 \times 10^{-3})</td>
</tr>
<tr>
<td>Norm. ( x(y) ) emittance ( \epsilon_{x(y)} )</td>
<td>(2.3(1.6)) mm mrad</td>
</tr>
<tr>
<td>Ave. ( x(y) ) beta function ( \beta_{x(y)} )</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Undulator period ( \lambda_u )</td>
<td>2.8 cm</td>
</tr>
<tr>
<td>Undulator parameter ( K )</td>
<td>1.31</td>
</tr>
<tr>
<td>Radiation wavelength ( \lambda )</td>
<td>530 nm</td>
</tr>
<tr>
<td>Max. radiation energy ( E_r )</td>
<td>31.5 ( \mu )J</td>
</tr>
<tr>
<td>Measured gain length ( L_g )</td>
<td>92 cm</td>
</tr>
</tbody>
</table>
scheme effectively preserves the narrowband radiation nominally produced in a standard non-chirped single-spike SASE FEL as seen in Ref. 10. However, it should be noted that no direct time-domain measurement had been achieved in Ref. 10 and the pulse length at the Fourier limit was only estimated from the spectral width. The TG FROG was thus implemented as an advanced diagnostic that could simultaneously perform the measurements, while pushing the diagnostic paradigm to shorter wavelength operation. Undulator tapering has also been studied previously in the context of seeded FELs by Wang et al.,25 where significant efficiency enhancement and spectral narrowing were observed. The present experiment, however, uses an undulator taper in a SASE FEL to suppress radiation growth where the change in resonance due to slippage is not compensated by an e-beam energy chirp. Superradiant FEL pulses generated by a coherent seed, such as those observed with a SHG FROG by Watanabe et al.26 also share many similar characteristics to single-spike SASE pulses. The physical mechanisms that govern their growth and propagation, however, are different than those at play here. Finally, we note that use of TG FROG, introduced here for a FEL, unambiguously yields the direction of time.

A typical experimental FROG trace is shown in Fig. 2(a); note that there is notable noise in the image. One strength of the reconstruction process, however, is the ability of the algorithm to filter out the noise because of the robust over-sampling involved.27 The reconstructed trace, Fig. 2(b), shows excellent overall agreement with the experimental trace. These results are compared to fully time-dependent, 3D particle-based simulations using GENESIS. Here, the experimentally measured e-beam parameters from Table I were used to specify the input particle distribution. Post-processing the simulation data with an algorithm to extract the light’s longitudinal profile at the transverse location indicated by the input mask in Fig. 1 results in the simulated FROG trace found in Fig. 2(c). The agreement in the details of the reconstructed and simulated FROG traces is striking. There are several distinct features which are well reproduced by the simulated FROG trace, such as the asymmetry in the main body, indicating a slight chirp, as well as the length and shape of the leading and trailing tails. This agreement extends to the reconstructed and simulated longitudinal profiles found in Figs. 3(a) and 3(b). The reconstructed longitudinal profile is composed of a dominant coherent spike with a full width at half maximum (FWHM) of $\delta \tau_{\text{FWHM}}^R = 98$ fs followed by a weaker trailing tail. The same signature can be found in the simulated results where the single dominant coherent spike has a FWHM of $\delta \tau_{\text{FWHM}}^G = 91$fs. It is interesting to note that there remains a small positive chirp in the phase in both profiles that is likely a result of the discrete nature of the tapering.

One advantage of using the FROG technique is that the temporal and spectral content along with the pulse’s phase information can be extracted simultaneously from the associated trace. This allows a rigorous determination of the time-bandwidth product (TBP),28 $TBP = \tau_{\text{rms}} \omega_{\text{rms}}$, where $\omega_{\text{rms}}^2 = \int_\infty \omega(t)^2 \cdotp E(t)^* \cdotp E(t) \cdotp \omega(t)^* \cdotp \omega(t) \cdotp \tau_{\text{rms}}$. Here, $E(t)$ is the electric field’s real amplitude, $\omega(t)$ is the temporal phase minus the mean frequency, the prime indicates differentiation with respect to time, and the intensity, $I(t) = |E(t)|^2$, is normalized to have unity time integral. This expression for the rms bandwidth contains contributions from variations in pulse amplitude as well as those due to phase changes. A

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**FIG. 2.** Comparison of the (a) experimental, (b) reconstructed, and (c) simulated FROG traces.

**FIG. 3.** Normalized longitudinal profiles from the (a) reconstruction and (b) simulation. (c) Comparison between the reconstructed intensity (solid-blue), simulated power at the mask (dashed-green), total pulse simulated power (dotted-red), and total pulse simulated power in the absence of an undulator taper (dotted-dashed-turquoise).
similar relationship holds for $\tau_{rms}^2$. For the pulse in Fig. 3(a), we have $TBP \sim 1.2$. This is highly significant, indicating that single-spike radiation production was nearly achieved while operating in the chirp and taper scenario.

One concern inherent with use of the TG FROG on a FEL is that the on-axis light is discarded by the input mask. Fig. 3(c) compares the off-axis and total power from the FEL is that the on axis light is discarded by the input mask. While the full power profile shows a more pronounced tail, the main features of the pulse are still captured by the off axis data gathered by the FROG system. Fig. 3(c) also shows the full power profile for an identical simulation in the absence of an undulator taper to emphasize the utility of the tapering. Multiple radiation spikes are clearly evident. The subtlety of interpreting the results from the TG FROG is an inherent challenge in FELs (as opposed to standard lasers), due to their non-negligible spatial chirp. Nevertheless, the current measurement is seen to be robust.

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