Study of X-ray Harmonics of the Polarized Inverse Compton Scattering Experiment at UCLA

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Introduction

\[ a_0 = \frac{e \, A_0}{m_e \, c^2} \]

\( a_0 \) is the normalized amplitude of the vector potential of the incident laser field (just like \( K \), undulator parameter)

\[ a_0 = 0.85 \times 10^{-5} \, \lambda_0[m] \sqrt{I_0[W/m^2]} \]

\( I_0 \) is the intensity and \( \lambda_0 \) is the wavelength of the incident laser

\[ \omega = \frac{\omega_0 \, 2 \, \gamma_0^2 \, (1 + \beta_{z0})}{1 + \frac{a_0^2}{2} + \gamma_0^2 \, \theta^2} \]

Frequency of the scattered photons where \( \omega_0 \) is the frequency of the incident laser, \( \gamma_0 \) is the relativistic factor of electron beam, \( \beta_{z0} \) is component of the electron initial velocity in the direction of laser pulse. (\( \beta_{z0} = 1 \) for head on scattering and \( \beta_{z0} = 0 \) for 90° scattering.)
ICS Calculations

\[ N = \frac{\pi}{3} \alpha N_0 a_0^2 \left( \frac{1 + a_0^2}{2} \right) \left( 1 + \beta_{z0} \right) \left( 1 + \overline{\beta}_z \right) \]

The total number of photons radiated by a single electron

\[ N_0 = \frac{(1 + \overline{\beta}_z) c T}{\lambda_0} \]

Where \( N_0 \) is the number of periods of the laser field with which electrons interacts

\[ \overline{\beta}_z = \frac{\beta_{z0} - a_0^2 / 4 \gamma_0 h_0}{1 + a_0^2 / 4 \gamma_0 h_0} \]

the average axial electron velocity

\[ \alpha = 1/137 \]

Fine structure constant

\[ h_0 = \gamma_0 (1 + \beta_{z0}) \]

\[ T = \frac{1}{c} \min\left( \frac{L_0}{1 + \overline{\beta}_z}, \frac{2 Z_r}{\beta_z}, \frac{2 w_0}{\beta_{\perp0}} \right) \]

Interaction time

\[ Z_r = \frac{\pi w_0^2}{\lambda_0} \]

Rayleigh range where \( w_0 \) is the waist radius of the laser
Gaussian beam

\[ I(r) = I_0 e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2}, \]

\[ w = 2 \sigma \]

Radius of the beam

\[ w(z) = w_0 \sqrt{1 + \left[ \frac{z M^2}{Z_r} \right]^2} \]

\[ F_\# = \frac{1}{2 M^2} \sqrt{\frac{\pi Z_r}{\lambda_0}} \]  
F number is \( f/D \)  
\( \sim z/2w(z) \)
75 µm waist radius using an F3 and 50 µm waist radius using F2 focusing geometries are achieved experimentally in Neptune.

The M factor is estimated to be 1.93.

For F2 geometry the Rayleigh range is 0.75 mm. 500 GW laser yields an intensity of $1.25 \times 10^{16}$ W/cm² and $a_0 \approx 1$. Since $a_0$ is proportional to $\lambda_0$ wavelength it is advantageous to use CO$_2$ laser compared to YAG lasers.

For F3 geometry the Rayleigh range is 1.7 mm. 500 GW laser yields an intensity of $5.5 \times 10^{15}$ W/cm² and $a0 \approx 0.67$. 
Time duration of the scattered photons

For head configuration the pulse duration will be determined by either the electron bunch length if \( L_b < L_0, Z_r \) or by the pulse length of the laser pulse if electron bunch length is longer than the laser pulse length. So we can express the duration of the scattered photons as

\[
\tau_\gamma = \frac{1}{c} \min[ L_b, (4Z_r + L_0) ]
\]

For transverse scattering duration of the scattered photons is again determined by electron bunch length if electron is short or by transverse dimension and Rayleigh range of the laser pulse along with the pulse length if electron is long. Thus we can express duration of scattered photons as

\[
\tau_\gamma = \frac{1}{c} \min[ L_b, 2Z_r + L_0, L_0 + 2r_b ]
\]
Inverse Compton Scattering Experiment
Design Parameters

Electron and Laser Beam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Beam Energy</td>
<td>14 MeV</td>
</tr>
<tr>
<td>Beam Emittance</td>
<td>5 mm-mrad</td>
</tr>
<tr>
<td>Electron Beam Spot size (RMS)</td>
<td>25 µm</td>
</tr>
<tr>
<td>Beam Charge</td>
<td>300 pC</td>
</tr>
<tr>
<td>Bunch Length (RMS)</td>
<td>4 ps</td>
</tr>
<tr>
<td>Laser Beam size at IP (RMS)</td>
<td>25 µm</td>
</tr>
<tr>
<td>CO2 laser wavelength</td>
<td>10.6 µm</td>
</tr>
<tr>
<td>CO2 laser Rayleigh range</td>
<td>0.75 mm</td>
</tr>
<tr>
<td>CO2 laser power</td>
<td>500 GW</td>
</tr>
<tr>
<td>CO2 laser pulse length</td>
<td>200 ps</td>
</tr>
</tbody>
</table>
### Design Scattered Photon Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head-on scattering</td>
<td></td>
<td>Transverse scattering</td>
<td></td>
</tr>
<tr>
<td>Scattered photon wavelength</td>
<td>5.3 nm</td>
<td>Scattered photon wavelength</td>
<td>10.7 nm</td>
</tr>
<tr>
<td>Scattered photon energy</td>
<td>235.3 eV</td>
<td>Scattered photon energy</td>
<td>117.7 eV</td>
</tr>
<tr>
<td>Scattered photon pulse duration (FWHM)</td>
<td>10 ps</td>
<td>Scattered photon pulse duration (FWHM)</td>
<td>10 ps</td>
</tr>
<tr>
<td>Interaction time</td>
<td>5 ps</td>
<td>Interaction time</td>
<td>0.33 ps</td>
</tr>
<tr>
<td>Number of periods that electrons see ($N_0$)</td>
<td>283</td>
<td>Number of periods that electrons see ($N_0$)</td>
<td>10</td>
</tr>
<tr>
<td>Number of photons emitted per electron ($N$)</td>
<td>3.34</td>
<td>Number of photons emitted per electron ($N$)</td>
<td>0.11</td>
</tr>
<tr>
<td>Total number of photons</td>
<td>$6.3 \times 10^9$</td>
<td>Total number of photons</td>
<td>$2.1 \times 10^8$</td>
</tr>
<tr>
<td>Half Opening Angle</td>
<td>2.7 mrad</td>
<td>Half Opening Angle</td>
<td>15 mrad</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10 %</td>
<td>Bandwidth</td>
<td>10 %</td>
</tr>
</tbody>
</table>
Nonlinear Harmonics

\[ \frac{\Delta \omega}{\omega_n} = \frac{1}{n N_0} \]

Bandwidth where \( N_0 \) is the number of periods that electrons interact. 10% bandwidth is estimated for the fundamental.

\[ \Delta \theta = \sqrt{\frac{(1 + a_0^2 / 2) \Delta \omega}{\gamma^2 \omega_n}} \]

Half opening angle of the harmonics. 15 mrad half angle is estimated.

\[ \omega_n = \frac{\omega_0 n 2 \gamma^2 (1 + \beta_{z0})}{1 + \frac{a_0^2}{2} + \gamma^2 \theta^2} \]

Frequency of harmonics

\[ \lambda_n = \frac{\lambda_s}{n} \]

Wavelength of harmonics
Double Differential Spectrum of Nonlinear Harmonics

\[
\frac{d^2 I_n}{d\omega d\Omega} = \frac{e^2 k_0^2}{\pi^2 c} \left[ \sin \frac{k \eta_0}{k} \right]^2 \left[ A_1 J_n^2 + A_2 \left( J_n' / b_t \right)^2 \right] \]

\[
\frac{d^2 I_{\theta n}}{d\omega d\Omega} = \frac{e^2 k_0^2}{4\pi^2 c} \left[ \sin \frac{k \eta_0}{k} \right]^2 \left[ k \frac{\partial g}{\partial \theta} \right] B_0 - \frac{\partial b_1}{\partial \theta} B_1 + 2 \frac{\partial b_2}{\partial \theta} B_2 \right]^2
\]

\[
\frac{d^2 I_{\phi n}}{d\omega d\Omega} = \frac{e^2 k_0^2}{4\pi^2 c} \left[ \sin \frac{k \eta_0}{k} \right]^2 \left[ \frac{\partial b_1}{\partial \phi} \right] B_1 - 2 \frac{\partial b_2}{\partial \phi} B_2 \right]^2
\]

\[
\frac{d^2 I_n}{d\omega d\Omega} = \frac{d^2 I_{\phi n}}{d\omega d\Omega} + \frac{d^2 I_{\theta n}}{d\omega d\Omega}
\]

Radiation Spectrum for circularly polarized laser beam*

Radiation Spectrum for linearly polarized laser beam*


Note: All the variables are defined in the paper
DDS plots for circularly polarized laser

n = 1

n = 2

n = 3

180° geometry

n = 1

n = 2

n = 3

90° geometry
DDS plots for linearly polarized laser

180° geometry

90° geometry parallel polarization

90° geometry perpendicular polarization
Four permanent magnet cubes (NdFeB) are positioned to produce Quadrupole field at the axis. Radia Program is used to design the magnet to produce 110 T/m gradient. Octagonal iron yoke provide proper field flow and hyperbolic iron tips produce perfect quadrupole field inside the magnet. The prototype was build and it is good agreement with the simulation. 1% field error in the magnetization of the cubes in worst orientation causes 10 µm axis offset which is quite reasonable. The cubes are measured and sorted to reduce possible errors.
Beam Transport for Inverse Compton Scattering

\[ \delta n / n_{sc} = 0. \]

Table name = TWISS
A Permanent magnet Dipole is designed to serve as energy spectrometer and dump for the electron beam. It bends the beam by 90°. The geometry is chosen so that beam always exits the dipole at 90° for various energies only with some offset.

Iron Yoke is designed for proper field flow
Magnets are made out of NdFeB high grade magnets which can yield 1.2-1.4 T magnetization.
The Magnetic field inside the gap is ~0.85 T. For 14 MeV energy design electrons the bend radius is about 55mm.

Field distribution inside the PMD gap simulated by Radia
Trajectory in the PMD

Trajectory of the 14 MeV electron beam in the PMD gap. Beam enters the magnet by 45° angle and exits by 45° angle. Y axis is the length of the magnet and x axis is width.

Synchrotron radiation wavelength is 7.6 µm

Angle of the trajectory

Angle linearly changes from $\pi/4$ to $-\pi/4$ inside the magnet.

$$\omega_c = \frac{3}{2} \gamma^3 \omega_0 = \frac{3}{2} \gamma^3 \frac{c}{\rho}$$

$$\lambda_c = \frac{4\pi}{3} \frac{\rho}{\gamma^3}$$
ICS Box design
Current Status

- PMQ design is complete and manufacturing is underway
- PMD design is complete and manufacturing is in progress
- Box design is in progress
- Soft X-ray camera may come from Argonne
- Polarization measurement is being researched