Coherent transition radiation diagnosis of electron beam microbunching

J. Rosenzweig *, G. Travish, A. Tremaine

Department of Physics, University of California, Los Angeles, 405 Hilgard Avenue, Los Angeles, CA 90024, USA

Received 10 April 1995

Abstract

The action of the free-electron laser instability (FEL) on an electron beam produces a longitudinal density modulation with a periodicity near the resonant radiation wavelength. This modulation, which can produce femtosecond or shorter microbunches inside of a macroscopic picosecond electron pulse, has been proposed for use as a prebunching injector for both higher harmonic FELs and short wavelength accelerators. Standard methods involving streak cameras or beam sweeping with dipole mode cavities will certainly fail to provide information about this longitudinal microstructure, however. In this paper we explore the use of coherent transition radiation generated from a foil at the exit of an FEL undulator to diagnose both the longitudinal and transverse electron microbunch structure.

1. Introduction

Coherent transition radiation has been recently introduced into the array of beam diagnostics [1–2] available for measuring the properties of picosecond or greater electron pulses. In this paper, we show how this technique may be extended to shorter time scales appropriate to the microbunches emerging from the exit of a free-electron laser undulator. This microbunching diagnostic provides information about the longitudinal dynamics of the electron beam in the FEL, which is critical to understanding the physics of high gain FEL systems. There are other applications of microbunched beams, e.g. the generation of short wavelength radiation using the harmonics of the microbunching is a popular feature in proposals for XUV and X-ray FELs [3]. Additionally, progress in short wavelength acceleration schemes, such as the plasma beatwave accelerator [4], the inverse Cherenkov accelerator [5], and the resonant dielectric structure accelerator [6] demand that this microbunching process be reliably quantified.

In order to understand the expected photon spectrum from coherent transition radiation due to a microbunched beam passing through a metal–vacuum boundary, we break down the analysis as follows: (i) transition radiation from a single electron, (ii) coherent transition radiation, (iii) coherent transition radiation from a microbunched electron beam, and (iv) comparison to the FEL spectrum.

2. Transition radiation

The theory of single particle transition radiation (TR), as well as its history of application in particle beam diagnosis, is well developed [1,2,7]. For a highly relativistic electron, the differential emission spectrum (energy per unit solid angle per unit frequency) is given by

$$\frac{d^2U}{d\omega \, d\Omega} \approx \frac{e^2}{4\pi^2c} \frac{\sin^2(\theta)}{(1 - \beta \cos(\theta))^2}$$

$$\approx \frac{e^2}{\pi^2c} \frac{\theta^2}{(\theta^2 + \gamma^{-2})^2},$$

where $\gamma^{-2} = 1 - \beta^2$, and $\beta = v/c$.

It is often remarked that this distribution is peaked off-axis at the approximate angle $\gamma^{-1}$. This is a bit misleading, however, since the angular distribution is weighted by an additional factor of $\sin(\theta)$. The expected number of photons per unit angle within a 1% bandwidth derived from this expression for a 12.5 MeV beam, such as we expect for the initial IR FEL experiments at UCLA [8], is shown in Fig. 1.

Note that there are not many photons ($\sim 10^{-4}$) per electron in a 1% bandwidth, they have a very large angular spread, and to collect even half of them one must have a
5.0
1.0
0.0
lW
0.0
n
10 20 30 40 50 60
(degrees)

Fig. 1. Number of transition radiation photons per electron in 1% bandwidth per unit angle for UCLA IR FEL energy, $\gamma = 25$.

15° acceptance. It will be seen below that the coherent portion of the TR spectrum displays the opposite characteristics: narrow bandwidth and angular spread, and, potentially, a much larger number of emitted photons.

3. Coherent transition radiation

The transition radiation from a bunched distribution of charges displays collective effects which can enhance the total number of photons radiated, as well as affecting the frequency and angular spectrum. This effect is due to the possibility of the emitted radiation from separate electrons being in phase with each other. To quantify this statement crudely before attempting a rigorous analysis, the number of electrons located within a longitudinal half-wavelength and a transverse half-wavelength divided by the Lorentz factor of each other, $N_c$, can radiate coherently, multiplying the emitted TR power by approximately $N_c$.

Before introducing the mathematical machinery needed to analyze this problem, we should discuss the electron distribution expected at the exit of the undulator. This subject will be amplified further in Section 4. We assume a beam which is initially a smooth bi-Gaussian in $r$ and $z$, and then forms microbunches under the action of the free-electron laser instability. These assumptions yield a modulated electron distribution which has the form

$$f(r, z) = N_b g(r) h(z).$$

where $N_b$ is the number of electrons per bunch and $k_r$ is the wave number of the fundamental FEL radiation. The bunching factors $b_n$ are in general near unity only when the FEL is near saturation, and when this is not the case, only the bunching on the fundamental is significant. Distributions of this sort may or may not be found in present day advanced accelerator experiments [4-5], but they must be found in a high gain free-electron laser experiment [8]. We therefore concentrate on the FEL case from this point on in this paper.

Coherent transition radiation (CTR) can be calculated in a straightforward manner, following the treatment by Shibata et al. [2] and Wartski et al. [7]. The differential radiation energy spectrum due to multiparticle coherence effects is represented by

$$\frac{d^2U}{d\omega d\Omega} \equiv N_b^2 F(\omega) \frac{d^2U}{d\omega d\Omega}_{\text{single } e^-}.$$  \hspace{1cm} (2)

where $F(\omega, \theta) = |f(\omega, \theta)|^2 = \int \int g(r) h(z) \exp(-ik \cdot x) \, d^3x$, and we have ignored the divergence of the beam, as it is generally small with respect to the emission angles of the CTR. This modified Fourier spectrum of the bunch can then be divided into two components, transverse and longitudinal, as $F(\omega, \theta) = F_T F_L$. The transverse integral is given by

$$F_T = \left| \int \int g(r) \exp(-ikr \sin(\theta) \cos(\phi)) \, dr \, d\phi \right|^2 = \exp(- (k \sigma_r \sin(\theta))^2),$$ \hspace{1cm} (3)

where $k$ is the radiation wave number. This factor serves to peak the coherent radiation in the forward direction at shorter wavelength, by phase differences simply related to the transverse geometry.

The longitudinal spectral factor gives, integrating over the microbunched distribution given in Eq. (1),

$$F_L = \left| \int h(z) \exp(-ikz \cos(\theta)) \, dz \right|^2 = \exp\left(- \frac{k(\sigma_z)^2}{2} + \sum_{n=1}^\infty b_n \exp\left(- \frac{(k-nk_r)^2 \sigma_z^2}{2}\right) + \exp\left(- \frac{(k+nk_r)^2 \sigma_z^2}{2}\right) \right|^2.$$ \hspace{1cm} (4)

This distribution displays peaks at all of the Fourier components of the bunch current. The peaks are narrow compared to their separation ($k_r \gg \sigma_z^{-1}$), and so the sum in Eq. (4) can be approximately squared as follows,

$$F_L \equiv \sum_{n=-\infty}^\infty b_n^2 \exp\left(- (k-nk_r)^2 \sigma_z^2\right).$$ \hspace{1cm} (5)
For now, let us concentrate on the radiation component at the FEL fundamental frequency, \( n = 1 \). We would like to know how many CTR photons are in the narrow band around this frequency, and in order to do so, we must integrate the differential spectrum

\[
\frac{d^2N}{dk d\theta} = \frac{\alpha N_b^2}{2\pi k} \frac{\sin^3(\theta)}{(1 - \beta \cos(\theta))^2} \times b_1 \left[ \exp\left(-\left(k\sigma_\gamma\sin(\theta)\right)^2\right) \right] 
\times \left[ \exp\left(-\left(k - nk_b\right)^2\sigma_\gamma^2\right) \right].
\]

The integral over the narrow (approximately 1%) frequency band is trivial,

\[
\frac{dN}{d\theta} \approx \frac{\alpha (N_b b_1)^2}{\sqrt{8\pi k\sigma_\gamma}} \frac{\sin^3(\theta)}{(1 - \beta \cos(\theta))^2} \times \left[ \exp\left(-\left(k\sigma_\gamma\sin(\theta)\right)^2\right) \right].
\]

The dependence of the radiation on the geometric factors is somewhat stronger than one might have anticipated. The expected dependence on the number of electrons which can radiate coherently is proportional to \( N_b b_1^2 \), but this must also be multiplied by the factor arising from the integral over the inherent angular dependence of the TR itself, which is approximately \( \gamma^2 / k^2 \sigma_\gamma^2 \). In the case of the UCLA IR FEL, this is a number on the order of \( 10^{-2} \). It should be noted that this number has in fact been assumed to be small in the series expansion of the exponential integral above, and that care must be taken to ensure that this series converges numerically.

4. Estimation of microbunching in a SASE FEL

In the limit that the one-dimensional theory of the FEL is valid, the bunching factor is simply related to the FEL power

\[
b_1 \approx \sqrt{P_{\text{FEL}} / \rho P_b},
\]

where \( P_b = eN_b\gamma m_e c^3 / \sqrt{2\pi} \sigma_\gamma \) is the electron beam power, and the dimensionless gain parameter is \( \rho = |g_{\omega} k_p / 4 k_w|^2 \).

\[
\rho = \left[ a_w k_p / 4 k_w \right]^{1/4},
\]

with \( k_p = \sqrt{4\pi \sigma_n \gamma^3} \), the relativistically correct beam plasma wave number. The FEL SASE power in this limit is given by the expression

\[
P_{\text{FEL}} = P_0 \exp\left(L_u / L_g - 1\right),
\]

where \( L_u \) is the undulator length, the gain length is given by

\[
L_g = L_u / 4\pi \sqrt{\rho},
\]

and the startup power is approximated as the spontaneous emission power within the FEL gain bandwidth and angular acceptance emitted with the first gain length [10]

\[
P_0 \approx \sqrt{\frac{\pi}{6} \frac{r_s (cB_w)^2 eN_b \gamma^5}{8m_e c k^2 \sigma_\gamma^2 \gamma^2}}.
\]

This model does not contain the two- or three-dimensional effects [11] such as diffraction, and lack of transverse spatial coherence, which generally increase the gain length and lower the bunching as well as causing more transverse

Fig. 2. Angular distribution of coherent transition radiation spectrum for UCLA IR FEL parameters: \( N_b = 6 \times 10^9 \), \( \sigma_\gamma = 700 \mu\text{m} \) and \( \sigma_\epsilon = 300 \mu\text{m} \), \( b_1 = 0.02 \) and \( \gamma = 25 \).
Fourier structure in the bunch current. As an example of the lowered bunching factor, in the UCLA IR FEL, with undulator wavelength \( \lambda_w = 1.5 \) cm, \( a_w = eB_x \lambda_w / 2 \pi m_e c^2 = 1 \), and length \( L_u = 60 \) cm, the bunching factor is calculated to be \( b_1 = 0.007 \), while three-dimensional TDA simulations give a more modest \( b_1 = 0.0018 \). Use of the TDA estimate gives \( N \approx 2 \times 10^7 \) CTR photons within the fundamental band of the FEL \( (k \approx k_w) \). This can be easily compared to the FEL photon number, as

\[
N_{\text{FEL}} = \frac{2 \pi \sigma_\perp}{\hbar c^2} P_{\text{FEL}} = \frac{b_1^2 \rho}{\hbar c} N_e \gamma m_e c^2, \tag{14}
\]

and so the ratio of the CTR to FEL photon number is

\[
\frac{N}{N_{\text{FEL}}} = \frac{r_c N_b}{4 \sqrt{\pi} \rho \sigma_\perp (k, \sigma_\parallel)^4}. \tag{15}
\]

This ratio would appear to favor higher energies, but since the factor \( k, \sigma_\parallel \propto \gamma^{3/2} \) due to the energy dependence of the beam emittance and the FEL resonant wavelength, the opposite is in fact true. The CTR and FEL power will be roughly equal in the UCLA IR FEL for an energy of approximately 10.5 MeV (34 \( \mu \)m radiation).

5. Experimental plans

Because the coherent enhancement of the TR spectrum at the exit of an FEL favors low energy operation, this effect should be straightforward to observe after the UCLA IR FEL undulator, given that (i) the number of photons to detect is nearly the same for the CTR and the FEL, and (ii) the angular spread of the CTR photons \( (\theta_{\text{CTR}} = (\sqrt{2} k_w \sigma_\parallel)^{-1} \approx 9 \) mrad) is also nearly the same as the diffraction limited FEL beam \( (\theta_{\text{FEL}} = (2k_w \sigma_\parallel)^{-1} \approx 6 \) mrad). Thus, one needs only insert a metal foil in the path of the beam which blocks the path of the FEL radiation, as shown in Fig. 3, in order to measure the CTR. A simple arrangement of collecting optics and a detector which is sufficient for initial measurements of the FEL output will also be adequate for observation of CTR.

It should be emphasized that this estimate of the angular spread in the CTR radiation ignores the effects of transverse spatial incoherence of the FEL radiation and, by extension, the bunching. In fact, the coherence of the FEL radiation should never be complete (basically only the fundamental Gaussian mode contributing to the FEL output) for \( L_e \theta_{\text{FEL}} \leq \sigma_\perp \), as the radiation in transversely adjacent portions of the beam cannot diffract out in sufficient time to affect the gain across the entire beam. In the UCLA case at 12 MeV, we expect nearly single transverse mode operation \( k, \sigma_\parallel \approx k_w \), while at higher energy, other transverse modes will enter into consideration. This effect should be observable through the angular distribution of the CTR.

6. Discussion

While the bunching factor \( b_1 \) on the radiation fundamental is important for advanced accelerator microbunch injectors, the bunching factor of the higher harmonics is more important for short wavelength FEL applications. The CTR scaling given above is not encouraging as far as measuring these parameters, however, since the number of photons \( \sim b_1^2 / n^5 \). In addition, much of the interest in
harmonic generation is at short wavelength (high energy), where the CTR flux is low even at the fundamental. Thus one should consider this a good tool only for diagnosing FEL bunching at scaled low energy, long wavelength experiments, such as the UCLA IR FEL. The quadratic dependence on $b_1$ of CTR signal means that small bunching will be difficult to measure. This signal can be enhanced, of course, by taking advantage of the phase space correlations which develop before the bunching in the FEL. If one passes the beam through a longer wavelength chicane after the undulator, the bunching can in principle be enhanced, and the CTR signal raised. This method can be thought of as a CTR optical klystron [12].

In contrast to FEL induced bunching, laser accelerators will display bunching at the applied, energy independent laser (fundamental or beatwave) wavelength, which is generally between 1 and 100 μm, and therefore the CTR signal will actually be larger at higher energy operation due to adiabatic damping of the transverse emittance. Thus this diagnostic scheme may be of broad use in short wavelength accelerators.

Acknowledgements

This work was performed with partial support from US Dept. of Energy grants DE-FG03-90ER40796 and DE-FG03-92ER40693, and the Alfred P. Sloan Foundation grant BR-3225.

References